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Determination of carbon emissions from hybrid road vehicles by mathematical modelling

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Abstract. This scientific paper presents an own research and an original point of view on the technical, mathematical and economical acceptance of road vehicle emissions. Specialists argue that practical experimentation through active type determinations can bring remarkable results in any field. In order to demonstrate practically and mathematically the current claims of the specialists, the present paper presents the actual results of some experiments obtained and some considerations on the mathematical model used for the validation of the objective functions, i.e. the CO_2 emissions obtained experimentally with specific measuring accuracy, on vehicles with hybrid propulsion. The proposed research also represents the authors' point of view, which is supported by the proposals made by the World Health Organization to reduce the speed of traffic in urban areas from 50 km/h to 30 km/h, with the aim of reducing the number of traffic accidents, as well as pollution with chemical and noise noxious substances and vibrations. The results of the tests carried out on urban roads were supplemented by the results of the tests carried out on motorways outside urban areas for comparative purposes, applying defensive driving methods (eco-driving). Thus, the paper presents the mathematical model and its validation applied on carbon dioxide (CO_2) emissions obtained under real driving conditions of road vehicles. Finally, some conclusions and research directions of the modelling are presented.

Keywords: *hybrid vehicle; chemical pollutants; experimental research; measuring instrument; mathematical model.*

Introduction

According to the European Parliament, 72% of CO_2 comes from car transport and efforts to improve fuel efficiency have been reduced [1]. Based on this data, things are not good at the moment because these aspects have not improved but have evolved in a negative direction, as demonstrated by the high number of thermal engine vehicles circulating in the world and polluting. It can be said that the culprit for air pollution, excluding industry but also other transport systems, is the road car system, which uses carbon-based fossil fuels. As a result of the combustion process in internal combustion engines, they release gases into the atmosphere that pollute the atmosphere. The transport industry's consumption of fossil fuels is second only to the manufacturing industry at 50%, with CO_2 emissions accounting for 25% of the total chemical pollutant emissions from motor vehicles into the atmosphere

[2]. Combustion gases [3], [4], interact and combine with the gases that make up atmospheric air, form harmful, dangerous substances, cause global warming and create the greenhouse effect. Ozone depletion and global warming are the effect of carbon monoxide and hydrocarbon pollution which increase when car engines are idling, at traffic lights or in gridlocked traffic in big cities, while nitrogen dioxide emissions become more pronounced when the car starts up [5, pp. 91-92]. It is well known that in the world's major cities motor vehicles are the main source of chemical and allergen pollution, noise and vibrations that affect the daily life of human habitats. The main sources of automotive pollutants are caused by incomplete or defective combustion of fuel, defective sealing of the piston-cylinder assembly, the fuel tank and fuel lines throughout the fuel line. Timely technical maintenance of the engine and other systems or installations increases the level of exhaust gas pollution resulting from the combustion of fossil fuels in cylinders, which are responsible for 85% of CO_2 emissions and 64% of total greenhouse gas emissions [6]. Air pollution is the phenomenon whereby the chemical composition of the air is altered in the form of a change in the proportion of its constituents or the appearance of new components that take on forms harmful to biotopes, biodiversity, human health, flora and fauna.

Chemical noxides are toxic gases emitted by car engines during the transport process. The most important pollutants released into the atmosphere by combustion engines are carbon dioxide (CO_2), hydrocarbons (HmCn), carbon monoxide (CO), nitrogen oxides (NO), sulphur dioxide (SO_2), nitrogen and nitrogen compounds. This research was chosen because carbon dioxide (CO_2) is the most important gas removed by internal combustion engines in cars. After water vapour, it is considered the second most important gas contributing to the greenhouse effect, along with methane gas (CH_4), nitrous oxide (N_2O), fluorinated gases (sulphur hexafluoride, hydrofluorocarbons and perfluorocarbons). The greenhouse effect is the phenomenon produced by gases in the earth's atmosphere that allow the sun's rays to pass through and reach the earth's crust, which traps heat (infrared radiation from the sun's rays). In turn, the earth's crust (the earth's surface) radiates some of the heat from the sun's rays back into the atmosphere, but it is trapped by water vapour and gases near the atmospheric layers, giving rise to the so-called greenhouse effect [7, p. 107]. The consequences of the greenhouse effect consist of visible changes in natural phenomena which have direct effects on the evolution of ecosystems through a global average temperature increase of 0.5ºC over the last 100 years. This has had the effect of raising sea and ocean levels by 20-25 centimeters as a result of melting glaciers and frozen ground in the northern part of the earth. The increase in global temperature is also easily seen in the increase in water input Earth-atmosphere-rivers, seas and oceans, defined by huge amounts of water from rain or snowfall that cause flooding, erode soil, scar human settlements, cause the disappearance of flora and fauna in areas formed over hundreds or thousands of years. Carbon dioxide is a natural by-product of human activities (a natural anthropogenic product of human or plant respiration, decay of organic matter, burning of fossil fuels or volcanic eruptions), is often used as a yardstick and accounts for 65% of total greenhouse gas emissions in the earth's atmosphere, being the most abundant. One molecule of CO_2 has a lifetime of 300 to 1,000 years. More than 50 billion tonnes of CO_2 are released into nature every year from the transport industry and power generation, accounting for about 1/3 of total annual CO_2 emissions. From the data researched, it was found that the level of environmental pollution with carbon dioxide and other harmful gases in the world has continued to increase. In this field, more than 200 scientific papers have been published in international journals dealing with CO_2 emissions from various sources [8]. Transport, as a branch of the world's industries, emits huge amounts of pollutants into the atmosphere, which attract and trap heat. In this context, if projected for the coming decades, it is argued that the demands on human mobility are expected to increase worldwide as the global population continues to grow, incomes rise, living standards rise, people's appetite for travel increases and the car becomes their number one favourite, becoming more and more indispensable to their transport needs [9].

The motivation of the scientific research in this scientific paper defines the main objective of the research, which starts from a high level of chemical and noise pollution of the environment. On the occasion of Global Road Safety Week, the Director of the World Health Organisation (WHO), Dr

Tedros Adhanom Ghebreyesus, proposed that the speed of road vehicles in cities around the world should not exceed 30 km/h. For cities in US countries, which use the imperial mile as a measure of distance, speeds should be limited to 20 miles, or 32.2 km/h [10]. European Parliament rapporteur Elena Kountoura called on European decision-makers to take the necessary measures to reduce the level of chemical pollutants in Europe's big cities and to impose a 30 km/h speed limit, together with the frequent use of environmentally friendly means of transport and a new European Road Transport Agency [11]. In this sense, in order to prevent exhaust gas, noise and vibration pollution, many European countries, including our country, have already approved and adopted, in whole or in part, measures to reduce the speed of road vehicles to 30 km/h on city streets. The advantages of reducing the speed of road vehicles in urban areas from 50 km/h to 30 km/h are as follows: fewer road accidents; fewer people killed or injured in road accidents; lower levels of chemical pollutants; lower levels of noise and noise pollution in road traffic; lower fuel consumption of vehicles. The disadvantages of reducing the speed of road vehicles in urban areas from 50 km/h to 30 km/h are as follows: traffic jams are created; public transport delays are created; stress and distrust of drivers, passengers and road transport users are increased. This research is intended to convey a different approach to mathematical modelling of chemical noxious parameters obtained through experimental research [12], [13], [14], *on combustion-engined cars by applying a mathematical model called the "Least Squares Method", which accurately returns the results obtained by actually measuring the amount of* CO_2 *from the combustion of a carbon-based fossil fuel at the tailpipe.*

1. The proposed research

The KANE AUTO 5-1 (Figure 1), a hand-held, Class 1 OIML, 5-gas, portable automotive emissions analyser, was used to measure the chemical noxides emitted from the car's spark ignition engines [15]. The main features of the KANE AUTO 5-1, Class 1 OIML, 5 gas automotive gas analyser are shown in Table 1.

The conditions under which the determinations were carried out to obtain experimental data on the exhaust gases emitted by the engine into the atmosphere are those used by major light-duty vehicle manufacturers for the approval of fuel consumption and pollutant emissions by the Worldwide Harmonized Light-Duty Vehicles Test Procedure (WLTP) [16], [17].

Figure 1. KANE AUTO 5-1 Portable Car Emission Analyzer OIML Class 1-5 gases [15].

Table 1. Technical characteristics of the KANE AUTO 5-1 Portable Analizer for car gas emissions, Class 1 OIML, 5 gases [15].

Measured parameters	Resolution of measurements [%]	Accuracy of measurements [%]	Measurement range [%]
Carbon Monoxide	0.01	±5 of reading*1	0-10

Measured parameters	Resolution of measurements [%]	Accuracy of measurements [%]	Measurement range [%]
(CO)		±0.06 from the volume *1	Over value: 20
Oxygen (O_2) (Fuel Cells)	0.01	±5 of reading *1 ±0,1 from the volume *1	0-25 Over value: 48
Hydrocarbons (HmCn) (HmCn-hexanes) (NDIR)	1 ppm	±5 of reading *1 ±12 ppm from the volume *1	0-3.000 ppm Over value: 10000 ppm
Carbon dioxide (CO_2) (infrared)	1 ppm	±5 of reading *1 ±0.5 from the volume *1	0-16 Over value: 25
Nitrogen oxide (NOx) (Fuel Cell)	1 ppm	0 - 4000 ppm ± 4 sau 25 ppm; 4000 – 5000 ppm ± 5%	0-5000 ppm
Engine oil temperature	0.1˚C/F	±.,0 ˚C ± 0,3% of reading ±3.6 ˚F ± 0,3% of reading	0 - 150 ˚C 32-302 ˚F
Rotations per minute (RPM)	1 rpm	50 rpm	200 - 6000 rpm
Carbon monoxide corrected CO	0.01	Calculating	0 - 15
Lambda AFR (petrol) (GPL)	0.001 00.01	-	0.8 – 1.2 11.76 – 17.64 12.48 – 18.72

*1 – use of dry gases at standard temperature and pressure;
*2 – standard for Auto 5-1 and 5-2 only.

To achieve the quoted specification, the instrument must be calibrated with fresh air (normally outside the workshop) at standard temperature and pressure (STP).

The type of calibration (Fresh Air Purge) is 90 sec. in automatic mode and 30 sec. in manual mode (Table 2, row 2).

It is possible to download this data by connecting the device via Bluetooth to a computer with KANE LIVE software installed [15]. This software allows the transfer of data stored in the memory of the device to the memory of the computer in an Excell file, from which values can be extracted, plotted 2 D and interpreted later.

All data was stored in the analyser memory and then downloaded. The data obtained from the experimental research on chemical noxious substances are reported to the current legislative provisions and were transferred to a PC, through which the experimental data were processed [18], using MODDE 13 software [19], Minitab 17 [20] and Excell.

Following the processing of the chemical and noise noxiousness data on PC (Personal Computer), the results were analysed and interpreted, based on which the conclusions, further research directions and databases for the mathematical model of the research were established.

During the experimental research the following conditions were observed:

- The test was carried out on routes composed of streets of all categories within Sibiu and on the A1 motorway, over distances of 5 - 20 km. This rule was respected because each street category has its own specificities (infrastructure, traffic speed, road signs, road traffic);
- Accelerations and decelerations were random, constant, in compliance with road traffic rules, preventive traffic rules and eco-driving. The rule is similar to the one used by major car manufacturers to approve fuel consumption and emissions from their internal combustion engines (WLTP method) [16], [17].

- Speed limits on urban streets: 30 km/h and 50 km/h respectively, and 130 km/h on motorways (subject to road traffic rules). The rule was requested by the WHO (World Health Organization);
- The vehicle was equipped with a portable chemical emissions measurement system (chemical NOx meter), which measures the amount of carbon monoxide (CO), hydrocarbons (HmCn), carbon dioxide (CO2), nitrogen oxide (NOx). A Kane Auto 5-1 portable gas analyser was used to measure the NOx emitted by the thermal engine of the Toyota hybrid car [15]. The gases were measured at the exhaust pipe outlet;
- Testing time: 10-15 minutes. At different speeds the time is variable;
- Outdoor ambient temperature: +20 to +25 °C. The optimum temperature for conducting experimental research is 22°C. At temperatures above 25°C, the Kane Auto 5-1 portable gas analyzer will stop working (the analyzer resets itself to zero);
- All auxiliary equipment that was switched on during the tests (air conditioning, headlights on, navigation system on, etc.) was taken into account. When electrical equipment is switched on, the electromagnetic field of the alternator and the mechanical resistance of the compressor in the air conditioning system increase the load on the engine and therefore the level of noxious emissions;
- The experimental investigations were carried out on the same urban routes (for speed = 50 km/h and for speed = 30 km/h) [10], [11]. The same routes were taken to meet the same traffic conditions, road traffic rules, travelling on the same road transport infrastructure.

It appears from the literature that there is no stipulated maximum reference value (Table 2) for CO_2 and NOx as regulated for CO and HmCn [21].

Table 2. Legal limit values for the main gases of exhaust for vehicles with spark ignition engines and compression ignition engines [21].

	CO (%)		CO_2 (%)		HmCn (ppm)		NOx (ppm)	
Vehicle type	**Operation in empty**	**2000-3000 rpm**	**Operation in empty**	**2000-3000 rpm**	**Operation in empty**	**2000-3000 rpm**	**Operation in empty**	**2000-3000 rot/min**
Vehicle registered up to 1986	4.5		Not specified		< 1000		Not specified	
Vehicle registered since 1987	3.5		Not specified				Not specified	
Vehicle with EURO 3-4 pollution standard*	0.5	0.3	Not specified		Not specified	**< 100**	Not specified	
Vehicle with EURO 5-6 pollution norm**	**0.3**	**0.2**	**Not specified**		**Not specified**	**< 100**	**Not specified**	

* For motor vehicles type-approved according to the limit values shown in row A or B of the table in section 5.3.1.4 of Annex I to Directive 70/220/EEC, as amended by Directive 98/69/EC (Euro 3 or Euro 4 motor vehicles of category M1, N1, M2 or N2).

** For vehicles type-approved according to Regulation (EC) No 715/2007 (Euro 5 or Euro 6 vehicles of category M1, N1, M2 or N2).

Carbon dioxide (CO_2) is a chemical that also results from the combustion of fossil carbon-based fuels in the cylinders of thermal engines. It is one of the most important harmful gases contributing to the greenhouse effect (global warming). It is also responsible for the contamination of certain acids, causing acidification of surface waters, especially seas and oceans.

Carbon monoxide (CO), from cars, is due to low combustion efficiency of heat engines. It has the property of accumulating in large quantities in winter and spring, when there is atmospheric calm and fossil fuel combustion in transport engines peaks. This usually occurs in large metropolitan areas where road traffic is heavy. The phenomenon also occurs because of a chemical stability that the gas has at low ambient temperatures. When carbon monoxide mixes with the organic compounds resulting from the incomplete combustion of fossil fuels in the cylinders of car engines (carbon and sulphur dioxide), the haze known as smog forms over large cities. This haze contains chemical compounds that are dangerous to human health.

Hydrocarbons (HmCn) are chemical compounds that contain molecules made up of carbon and hydrogen atoms, which come from the unburned fuel that is released into the atmosphere by heat engines. This type of substance has no immediate effect on human health when released into the environment, except for polycyclic aromatic hydrocarbons, which have a carcinogenic effect. Hydrocarbons emitted by the engines of means of transport form photochemical smog, which is actually a haze produced in the earth's atmosphere under the direct effect of the sun's rays. It is also made up of nitrogen oxides from the exhaust gases of cars. The final compound of these harmful gases, smog, has a direct effect on human health, irritating the eyes and mucous membranes, reducing visibility and becoming dangerous for road traffic, especially in large cities where traffic becomes heavy. Smog comes from the English word for smoke and fog.

Nitrogen compounds are a noxious substance resulting mainly from the combustion of diesel in internal combustion engines. It combines with water in the atmosphere to produce acid rain, which has destructive effects on buildings and vegetation. They are chemical pollutants that are a constant contributor to environmental pollution, and nitrogen dioxide (NO_2), a combination of nitrogen and oxygen, is considered one of the most dangerous pollutants. Nitrogen dioxide is formed at high temperatures and its source is carbon-based fuel burned in internal combustion engines of cars. Nitrous oxide (NO), which is released into the atmosphere, is also a major contributor to smog formation, along with hydrocarbons. They affect the health of living beings by having haemoglobin-binding effects, affecting the lungs and creating lung diseases. Nitrogen oxides and sulphur oxides contribute to the formation of acid rain which has a harmful effect on humans, has a slow effect over time on art objects, buildings and biosystems in general, poisoning water, flora and fauna. Other harmful substances resulting from the motor vehicle transport process are: volatile organic compounds (VOCs) - benzene, soot, asbestos, dust particles (PM 2.5) [22], lead compounds, benz-α-pyrene, aldehydes and heavy metals.

2. Results and discussions

Table 3 shows the centralized results of the average values of CO_2 emissions obtained from the experiments carried out on the Toyota hybrid 2.5 liters, Euro 6 car, when driving in urban areas at a speed of 30; 50 and extra-urban areas at a speed of 130 km/h.

Table 3. The average values of CO_2 emissions obtained from the expeiments carried out on the hybrid Toyota car, when traveling the urban route at a speed 30 km/h, 50 km/h and extra-urban, at a speed of 130 km/h.

The type and make of the vehicle	Displacement (cm^3)	Kilometers on board (actual km)	Seniority (years)	Average CO_2 values (%)		
				Urban 30 km/h	**Urban 50 km/h**	**Extra-urban 130 km/h (highway)**

The type and make of the vehicle	Displacement (cm^3)	Kilometers on board (actual km)	Seniority (years)	Average CO_2 values (%)		
				Urban 30 km/h	**Urban 50 km/h**	**Extra-urban 130 km/h (highway)**
Toyota Euro 6 hybrid car	2,500	25,696	3	9.33	11.6	12.87

The CO_2 values in Table 3 were obtained by experimental research on the Toyota hybrid car and were measured with the KANE AUTO 5-1 portable gas analyser available (Figure 1). The results obtained are as follows (Table 3): when driving the urban route at a speed of 30 km/h, CO_2 reached an average value of 9.33%; when driving the urban route at a speed of 50 km/h, CO_2 reached an average value of 11.6%; when driving the motorway route at a speed of 130 km/h, CO_2 reached an average value of 12.87%.

Figure 2 shows the CO_2 evolution (%) for the Toyota hybrid Euro 6 car in urban (30 km/h, 50 km/h) and motorway (130 km/h) driving.

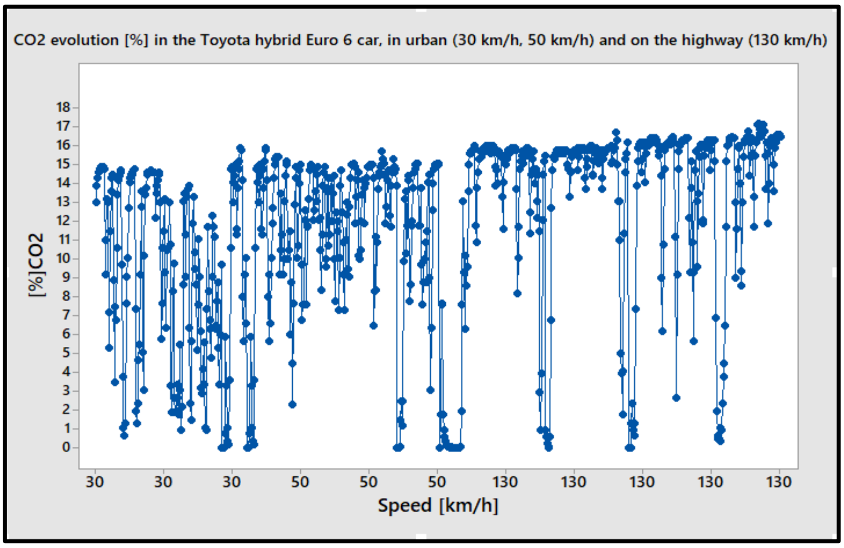

Figure 2. The evolution of CO_2 in the hybrid Toyota car in urban (30 km/h, 50 km/h) and extra-urban (130 km/h) areas.

The analysis of the experimentally obtained data shows a decreasing trend of CO_2 over the three routes, due to the reduction of the driving speed of the vehicle in the urban environment from 50 km/h to 30 km/h. In the extra-urban environment the CO_2 pollution level increased due to the increase in driving speed which, depending on the situation, required engine torque, speed and power. Because the experimental investigations were conducted at high ambient temperatures (25-30° C), due to the high temperature of the air admitted to the engine, it influenced the temperature level per cycle, influencing the fuel gas spray in the combustion chamber at the time of injection, as well as the combustion speed contributing to the formation of pollutant compounds [23].

In the following an experimental statistical modeling using an active type experiment is presented, and the representations of the evolution of the percentage of CO_2 are spatial, i.e. by contour lines obtained by a cross-section made by a spatial representation with a plane parallel to the base plane. The evolution of CO_2 in relation to other gases released into the atmosphere by the car's internal combustion engine is shown below.

a) Figure 3 shows the trend of CO_2 (%) versus CO (%) and HmCn (ppm) at 30 km/h, which is as follows:

- if CO_2 decreases to 8%, HmCn increases to 64.18 ppm and CO increases to 0.04%;
- if CO_2 increases to 18%, HmCn decreases to 18 ppm and CO increases to 0.11%.

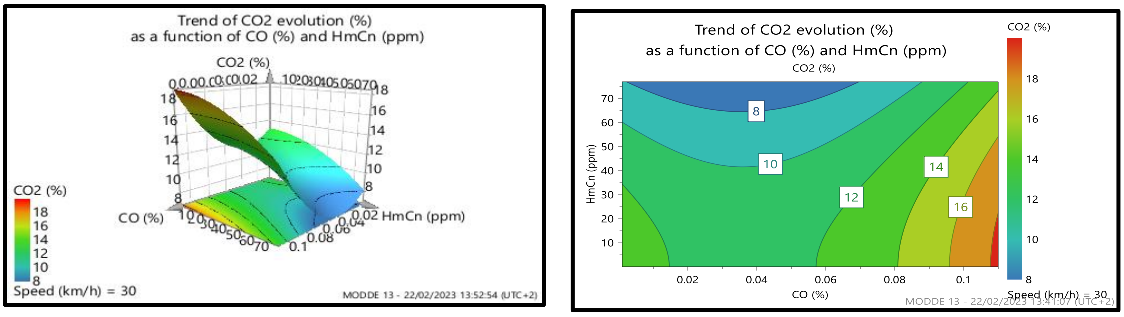

Figure 3. Trend of CO_2 evolution (%) as a function to CO (%) and HmCn (ppm) in the Toyota Euro 6 hibrid car, in the urban at a speed of 30 km/h.

b) Figure 4 shows the trend of CO_2 (%) versus CO (%) and HmCn (ppm) at 50 km/h, which is as follows:

- if CO_2 decreases to 0%, HmCn increases above the legal norm to 133.5 ppm and CO increases above the legal norm to 0.95%;

- if CO_2 increases to 30%, HmCn increases above the legal standard to 498.2 ppm and CO increases above the legal standard to 0.4%.

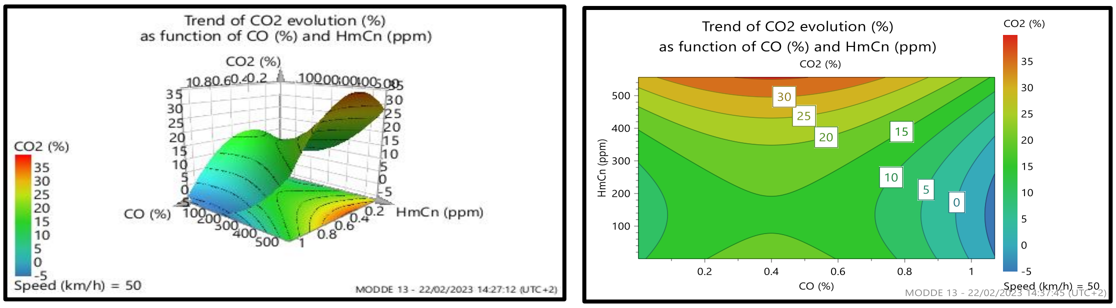

Figure 4. Trend of CO_2 evolution (%) as a function to CO (%) and HmCn (ppm) in the Toyota Euro 6 hibrid car, in the urban at a speed of 50 km/h.

c) Figure 5 shows the trend of CO2 (%) versus CO (%) and HmCn (ppm) at 130 km/h, which is as follows:

- if CO_2 decreases to 7%, HmCn increases above the legal norm to 311 ppm and CO decreases to 0.02%;

- if CO_2 increases to 16%, HmCn increases above the legal standard to 169 ppm and CO increases above the legal standard to 0.81%.

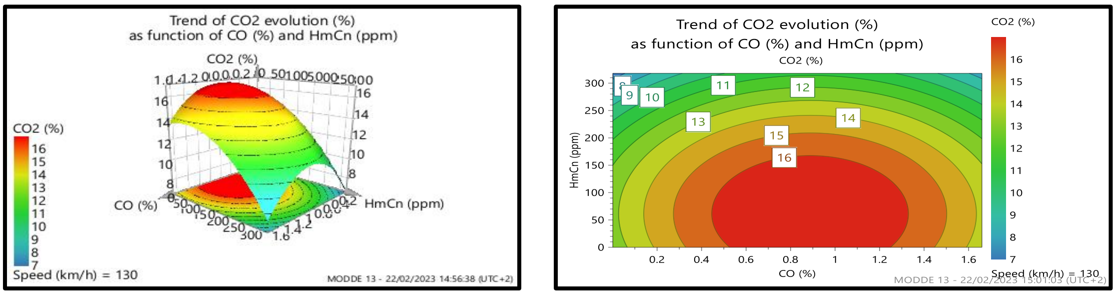

Figure 5. Trend of CO_2 evolution (%) as a function to CO (%) and HmCn (ppm) in the Toyota Euro 6 hibrid car, in the extra-urban at a speed of 130 km/h.

Analysis of the spatial plots shows that CO and HmCn increase above the legal norm in proportion to the increase in car speed. The increase in CO and HmCn occurs as a result of incomplete combustion, caused by the lack of oxygen in the mixture. In incomplete combustion of carbon-based fuels at high dosage (*power dosage* $\lambda p = 0.8\text{-}0.9$), fuel consumption increases due to incomplete combustion. The percentage of CO and HmCn increases when burning enriched mixtures. If air-fuel mixtures are enriched, for each 0.1 unit reduction of λ, the CO concentration increases by 3.5%. An operation of thermal engines in a high temperature environment influences the initial temperature increase of the combustion process, which leads to a detonation combustion and the release of CO and HmCn into the atmosphere. In the middle of hot summer days, when the ambient temperature reaches 30-40˚C, the frequency and intensity of detonation combustion in the cylinders of thermal engines is higher, compared to cooler days or at night when the ambient temperature is lower and the humidity is higher. However, more attention needs to be paid to this aspect as it has been experimentally demonstrated that high humidity of the air sucked in by engines (including supercharged engines), when using a petrol with octane number reduced by 8-15 units, at humidity variations of 10g/kg fuel, fuel consumption increases by 2.7% and its power decreases by 2%, generating excess CO (%).

At high engine speeds, the intake velocity increases and the flow turbulence in the intake manifold increases. A high engine speed decreases the duration of combustion in the cylinders, reduces the pressure regime that occurs in front of the combustion front and reduces the tendency for detonation. The consequence is a reduction of CO and HmCn. The idling regime of the thermal engine generates large amounts of CO due to the enriched mixture. The nature of the fuel also directly influences the initial and final phase of the combustion taking place in the engine cylinders. A low octane petrol creates detonation (formation of ignition nuclei in front of the combustion front) and this results in incomplete fuel burns and the release of unburned petrol, i.e. HmCn, into the atmosphere.

2.1 Possible causes that decrease or increase CO_2 emissions

The decrease or increase in the level of chemical noxious emissions from an internal combustion engine may be due to a number of factors that directly influence the operation and combustion of the cylinders. Table 4 shows the possible causes that decrease or increase the occurrence of CO_2 in internal combustion engines.

Table 4. Possible causes that decrease or increase the occurrence of CO_2 in internal combustion engines.

Name of the noxious substances	**Cause of occurrence**	**Mode of manifes-tation**		**Explanations, observations, comments**
		Decrease	**Increase**	
				A. Construction factors of heat engines
Carbon dioxide CO_2(%)	Use of the Atkinson principle [24], in the engine cylinder air intake	x	-	The principle was invented before the First World War by the British engineer James Atkinson, who applied his invention using certain mechanical elements on the Otto engine (an assembly of three articulated connecting rods, one of which has a hinged attachment point to the engine crankcase. The mechanism also has camshafts which, with the ability to alter the advance in rotation to delay the closing of the intake valves). The principle is much more efficient than the principle of operation of the Otto

Name of the noxious substances	**Cause of occurrence**	**Mode of manifes-tation**		**Explanations, observations, comments**
		Decrease	**Increase**	
				internal combustion engine [25, p. 64]. It consists of delaying the closing of the inlet valve at the end of time 1, the inlet. Thus, by means of the engine and exhaust gas distribution mechanism, modified and adapted for the engine cycle, the closing of the inlet valve on each cylinder is delayed and overlapped over the beginning of time No. 2, compression. Closing the intake valve with a delay causes some of the air sucked into the engine (the oxidant) to be exhausted by the piston by moving it from the Internal Blind Spot (IBS) to the External Blind Spot (EBS) of engine cylinder in the intake manifold at the start of the compression stroke. With a reduced amount of air in the cylinder, the Electronic Control Unit (ECU) receives a signal to dispense a reduced amount of fuel (relative to the amount of air in the cylinder) for injection, making combustion almost complete and efficient. The efficiency of the engine using the Atkinson principle is high, close to that of a diesel engine (35-45%), compared to a classic gasoline engine (Otto), which has a lower efficiency (25-35%). At low quantities of air-fuel mixture, there is always the problem of decreasing engine power. This is why it is necessary to make a certain compromise in the correct operation of the thermal engine, the principle not being used permanently during its operation. The internal combustion engine, which uses this principle, is used exclusively in hybrid vehicles and only operates when the speed is constant (not when starting or under load), in which case the electric motor(s) compensates and provides the power difference. In summary, the Atkinson principle reduces fuel consumption, greatly improves combustion in the cylinders (almost ideal combustion), the engine does not overheat, there is no high pressure force (compression ratio drops considerably from 16:1 to 12:1), and CO_2 pollution levels drop considerably, all of which is achieved on an aspirated petrol engine [26].
	B. Factors relating to vehicle operation			
	High temperature in the	-	x	CO_2 dissociation occurs at high combustion chamber temperatures. During the fourth stroke of the combustion engine, the exhaust, carbon monoxide

Name of the noxious substances	**Cause of occurrence**	**Mode of manifes-tation**		**Explanations, observations, comments**
		Decrease	**Increase**	
	engine combustion chamber			(CO) ignites in the exhaust column in the presence of oxygen (O_2), forming carbon dioxide (CO_2). In the presence of air or oxygen CO burns with a bluish flame, releasing a lot of heat and generating CO_2.

According to the data presented in Table 3, the factors influencing the occurrence of CO_2 in internal combustion engines are factors related to the construction of thermal engines and factors related to vehicle operation [3, pp. 885-913]. They are defined by the authors based on expert knowledge, literature and online.

2.2. The proposed mathematical model

The proposed mathematical model demonstrates what happens in the memory of the measuring device, which was used to make the actual, practical measurements in the experimental research on CO_2 noxides. The proposed mathematical model accurately validates the average values of the chemical noxiousness measured at the tailpipe (CO_2) with specific equipment. To begin with, a short introduction to linear algebra is proposed, presenting theoretical notions on the definition of vector space.

2.2.1 Theoretical notions

Definition of vector space [27, p. 1]:

Let V be a non-empty manifold and K a commutative body (field). A vector space structure on the manifold V, over the commutative body K, (a K -vector space) is defined by a triplet (V, +, $\cdot_{sc}$), where (V, +) is a group, and -sc : K ×V → V is an external composition law, so the properties hold:

(V1) $\alpha \cdot (\bar{x} + \bar{y}) = \alpha \cdot \bar{x} + \alpha \cdot \bar{y}$, $(\forall)\alpha \in K, \bar{x}, \bar{y} \in V$; (1)

(V2) $(\alpha + \beta) \cdot \bar{x} = \alpha \cdot \bar{x} + \beta \cdot \bar{x}$, $(\forall)\alpha, \beta \in K, \bar{x} \in V$; (2)

(V3) $(\alpha \cdot \beta) \cdot \bar{x} = \alpha \cdot (\beta \cdot \bar{x})$, $(\forall)\alpha, \beta \in K, \bar{x} \in V$; (3)

(V4) $1 \cdot \bar{x} = \bar{x}$, $(\forall)\bar{x} \in V$. (4)

The elements of the manifold V are called vectors, the internal composition law „+" on V-V→V is called vector addition, and the external composition law „-" on V is called scalar product. A vector space over the body of real numbers (K = R) is called a real vector space, and a vector space over the body of complex numbers (K = C) is called a complex vector space. Any vector space considered hereafter will be real or complex unless otherwise specified. Two vectors $\bar{x}$ and $\bar{y}$ for which there exists a scalar $\alpha \in K$ such that $\bar{x} = \alpha\bar{y}$ or $\bar{y} = \alpha\bar{x}$ are called collinear vectors.

If $\bar{x}_1, \ldots, \bar{x}_n \in V$ are vectors and $\alpha_1, \ldots, \alpha_n \in K$ are scalars, then the vector $\bar{x} = \sum_{i=1}^{n} \alpha_i \bar{x}_i$ is a said to be a linear combination of the vectors $\bar{x}_1, \ldots, \bar{x}_n$.

Thus, for example, if $\bar{x}, \bar{y} \in V$ și $\alpha, \beta \in K$, then the vector $\bar{z} = \alpha\bar{x} + \beta\bar{y}$ is a linear combination of vectors $\bar{x}$ și $\bar{y}$.

A crowd $\{\bar{x}_1, \ldots, \bar{x}_n\} \subset V$, is a basis of the vector space V if:

i) vectors $\bar{x}_1, \bar{x}_2, \ldots \bar{x}_n$ are linearly independent, i.e. from $\alpha_1 \cdot \bar{x}_1 + \ldots \alpha_n \cdot \bar{x}_n = \bar{0}$, where, ($\bar{0}$ = zero vector), are obtained $\alpha_1 = \alpha_2 = \ldots \alpha_n = 0$; $(0 \in K)$;

ii) $\{\bar{x}_1, \ldots, \bar{x}_n\}$ generates on V, i.e $(\forall)v \in V$, there is scalars $\alpha_1, \alpha_2, \ldots \alpha_n \in K$, so that $V = \alpha_1 \bar{x}_1 + \ldots \alpha_n \bar{x}_n$.

An function $T: V_1 \rightarrow V_2$, where V_1, V_2 are vector spaces over K, is called a linear application if:

$$T(\alpha_v + \beta_u) = \alpha T(v) + \beta\, T(u),\ (\forall)v,u \in V_1,\ (\forall)\alpha,\ \beta \in K. \qquad (5)$$

2.2.2 Mathematical model

We consider the vector space $(R^n,+,\cdot)$ where, $x + y = (x_1 + y_1, \ldots, x_n + y_n)$, (internal operation), $x \cdot x = (\alpha x_1, \ldots\ \alpha x_n))$, (external operation), $n \in N$, $n \geq 1$.

For the mathematical model the vector space is considered $(R^5,+,\cdot)$, which is the space determined by the car parameters $x = (x_1, x_2, \ldots, x_5)$, where x_1, x_2, x_3, x_4, x_5, means the speed of the vehicle, the European pollution standard, the cylinder capacity, the kilometres on board, the age in years, and the vector space $(R^4,+,\cdot)$, which is the space determined by the nox parameters $y = (y_1, y_2, \ldots y_4)$, where y_1, y_2, y_3, y_4, are the quantities of carbon dioxide (CO_2), or other gases: carbon monoxide (CO), hydrocarbons (H_mC_n) and nitrogen oxides (NOx).

Choosing bases E = {(1,0,0,0,0); (0,1,0,0,0); (0,0,1,0,0); (0,0,0,1,0); (0,0,0,0,1)} in R^5, respectively F = {(1,0,0,0,0); (0,1,0,0,0); (0,0,1,0,0); (0,0,0,1,0); (0,0,0,0,1)} in R^4. (6)

Any linear application $T:R^5 \rightarrow R^4$ is uniquely determined by its values in E-base vectors, the images of E-base vectors being uniquely expressed in terms of F-base vectors, i.e,

$$T(ei) = \sum_{j=1}^{4} a_{ij} f_j,\ i = 1,2,3,4,5 \qquad (7)$$

where,

$$e_1 = (1,0,0,0,0);\ e_2 = (0,1,0,0,0);\ e_3 = (0,0,1,0,0);\ e_4 = (0,0,0,1,0);\ e_5 = (0,0,0,0,1); \qquad (8)$$

$$f_1 = (1,0,0,0,0);\ f_2 = (0,1,0,0,0);\ f_3 = (0,0,1,0,0);\ f_4 = (0,0,0,1,0);\ f_5 = (0,0,0,0,1); \qquad (9)$$

For $x \in R^5$, $x = x_1e_1 + x_2e_2 + x_3e_3 + x_4e_4 + x_5e_5$, we obtained:

$$T(x) = T\left(\sum_{i=1}^{5} x_i e_i\right) = \sum_{i=1}^{5} x_i T(e_i) = \sum_{i=1}^{5} x_i \sum_{j=1}^{4} a_{ij} f_j \qquad (10)$$

$$T(x) = x_1(a_{11}, a_{12}, a_{13}, a_{14}) + x_2(a_{21}, a_{22}, a_{23}, a_{24}) + x_3(a_{31}, a_{32}, a_{33}, a_{34}) + x_4(a_{41}, a_{42}, a_{43}, a_{44}) + x_5(a_{51}, a_{52}, a_{53}, a_{54}) = \left(\left(\sum_{i=1}^{5} x_i a_{i1,}\right), \left(\sum_{i=1}^{5} x_i a_{i2,}\right), \left(\sum_{i=1}^{5} x_i a_{i3,}\right), \left(\sum_{i=1}^{5} x_i a_{i4,}\right)\right) = y \qquad (11)$$

For different values of speed x_1 minimum, average or maximum values of the noxious parameters are determined.

$$\begin{cases} x^1 = (x_1^1, x_2, \ldots, x_5) \text{ it follows that } T(x^1) = y^1 = (y_1^1, y_2^1\ y_3^1, y_4^1); \\ x^2 = (x_1^2, x_2, \ldots, x_5) \text{ it follows that } T(x^2) = y^2 = (y_1^2, y_2^2\ y_3^2, y_4^2); \\ x^3 = (x_1^3, x_2, \ldots, x_5) \text{ it follows that } T(x^3) = y^3 = (y_1^3, y_2^3\ y_3^3, y_4^3). \end{cases} \qquad (12)$$

From relations (11) and (12) we obtain:

$$\begin{cases} x_1^1\, a_{11} + x_2\, a_{21} + x_3\, a_{31} + x_4\, a_{41} + x_5\, a_{51} = y_1^1; \\ x_1^2\, a_{11} + x_2\, a_{21} + x_3\, a_{31} + x_4\, a_{41} + x_5\, a_{51} = y_1^2; \\ x_1^3\, a_{11} + x_2\, a_{21} + x_3\, a_{31} + x_4\, a_{41} + x_5\, a_{51} = y_1^3. \end{cases} \qquad (13)$$

$$x_2\, a_{21} + x_3\, a_{31} + x_4\, a_{41} + x_5\, a_{51} = y_1^1 - x_1^1\, a_{11} = y_1^2 - x_1^2\, a_{11} = y_1^3 - x_1^3\, a_{11} \qquad (14)$$

Immediate conclusion: It is found that the relations in (13) lead to the above equalities, which assign different values to a11, which is NOT TRUE, it is FALSE. Noxes parameters cannot be expressed linearly as a function of vehicle parameters (driving speed, European pollution standard, engine capacity, on-board mileage or vehicle age).

For the relationship to be TRUE, the quantities of pollutants will be determined as a function of speed, when the parameters x_2, x_3, x_4, x_5 remain unchanged, using adjustment elements of the experimental data. Therefore, the measured values in Table 5 we are considered:

Table 5. Measured values of speed (s) and noxiousness (y).

s - speed	s_1	s_2	s_3

$y = f(s)$, $f:[0, 130] \rightarrow R$;

y - quantity of noxious	y_1	y_2	y_3

Fix a certain class F of functions (trends) h:[0, 130] →R and is noted with $f^* = (f(s_1), f(s_2), f(s_3)$;
We consider the scalar product $(f^*, h) = \sum_{k=1}^{3} f(sk) \cdot h(sk)$ and the norm $||h|| = \sqrt{(h,h)}$; (15)
Adjusting numerical data $(s_k, f(s_k))$, k = 1,2,3 using F-class functions, is to determinate a function $h^* \in F$, so that:

$$||f^* - h^*|| \leq ||f^* - h||, (\forall|\ h \in F) \tag{16}$$

In general, the choice of trend is made by plotting the points $P_k(s_k, y_k)$, k = 1,2,3.
Example: for the Toyota RAV 4 hybrid car 2,5 liters, Euro 6 the average values of the CO_2 = y [%], and 30; 50; 130 = s (speed) [km/h], shown in Table 6, as follows:

Table 6. Average values of
CO_2 = y [%] and 30; 50; 130 = s (speed) [km/h].

v	30	50	130
y	9,33	11,6	12,87

The average values in Table 5 are shown in Table 2 for the Toyota RAV 4 hybrid car, Euro 6.
Figure 6 shows the CO_2 trend example obtained by plotting the points Pk(sk, yk), k = 1,2,3 for the Toyota RAV 4 hybrid car 2,5 liters, Euro 6 at 30 km/h, 50 km/h and 130 km/h for CO_2 emissions.

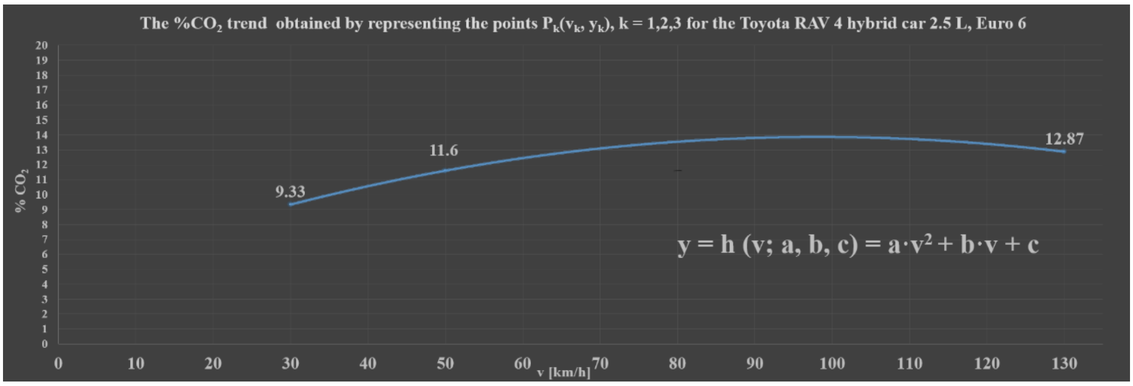

Figure 6. The $\%CO_2$ obtained by representing the points $P_k(s_k, y_k)$, k = 1,2,3 for the Toyota RAV 4 hybrid car 2.5 liters, Euro 6.

From the analysis of the data shown in Figure 6, it appears that the trend is parabolic, i.e:

$$y = h\,(s;\, a, b, c) = a \cdot s^2 + b \cdot s + c. \tag{17}$$

Determine the parameters a,b,c such that h (v; a, b, c) solves the minimum problem (16), is done by using ***the least squares method***, which consists of determining the parameters a, b, c such that the sum of $S(a,b,c) = \sum_{k=1}^{3} [h(s_k; a,b,c) - f(s_k)]^2$, be minimal. (18)
This function of three variables, has a single point of global minimum whose coordinates are determined using the following system:

$$\begin{cases} \frac{\partial S}{\delta a} = 2 \sum_{k=1}^{3} [h(s_k; a,b,c) - y_k] \cdot \frac{\partial h}{\partial a}(s_k; a,b,c) = 0 \\ \frac{\partial S}{\partial b} = 2 \sum_{k=1}^{3} [h(s_k; a,b,c) - y_k] \cdot \frac{\partial h}{\partial b}(s_k; a,b,c) = 0 \\ \frac{\partial S}{\partial c} = 2 \sum_{k=1}^{3} [h(s_k; a,b,c) - y_k] \cdot \frac{\partial h}{\partial c}(s_k; a,b,c) = 0 \end{cases} \tag{19}$$

$\frac{\partial s}{\partial a}$ is the partial derivative of the function S with respect to the variable a. Thus, by solving (19), we obtain:

$$\begin{cases} \sum_{k=1}^{3}(\mathrm{as}_k^2 + \mathrm{bs}_k + c - y_k) \cdot s_k^2 = 0 \\ \sum_{k=1}^{3}(\mathrm{as}_k^2 + \mathrm{bs}_k + c - y_k) \cdot s_k = 0 \\ \sum_{k=1}^{3}(\mathrm{as}_k^2 + \mathrm{bs}_k + c - y_k) \cdot 1 = 0 \end{cases} \tag{20}$$

Use the linearity of the sum and get:

$$\begin{cases} a\sum_{k=1}^{3} s_k^4 + b\sum_{k=1}^{3} s_k^3 + c\sum_{k=1}^{3} s_k^2 = \sum_{k=1}^{3} s_k^2\, y_k \\ a\sum_{k=1}^{3} s_k^3 + b\sum_{k=1}^{3} s_k^2 + c\sum_{k=1}^{3} s_k = \sum_{k=1}^{3} s_k\, y_k \\ a\sum_{k=1}^{3} s_k^2 + b\sum_{k=1}^{3} s_k + 3c = \sum_{k=1}^{3} y_k \end{cases} \tag{21}$$

For ease of calculation the following notations are made:

$$\sum_{k=1}^{3} s_k^4 = \alpha; \quad \sum_{k=1}^{3} s_k^3 = \beta; \quad \sum_{k=1}^{3} s_k^2 = \gamma; \quad \sum_{k=1}^{3} s_k = \theta; \tag{22}$$

$$\sum_{k=1}^{3} s_k^2\, y_k = e; \quad \sum_{k=1}^{3} s_k\, y_k = f; \quad \sum_{k=1}^{3} y_k = g. \tag{23}$$

Apply the notations in relations (22) and (23) and obtain the system of equations:

$$\begin{cases} \alpha a + \beta b + \gamma c = e \\ \beta a + \gamma b + \theta c = f \\ \gamma a + \theta b + 3c = g \end{cases} \tag{24}$$

Deduct from the system (24), the variables Δ_S , Δ_1, Δ_2, Δ_3 and obtain the mathematical relations, which will define the Cramer system:

$$\Delta_S = \begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \theta \\ \gamma & \theta & 3 \end{vmatrix} = 3\alpha\gamma + \beta\theta\gamma + \beta\gamma\theta - \gamma^3 - \alpha\theta^2 - 3\beta = 3\alpha\gamma + 2\beta\gamma\theta - \gamma^3 - \alpha\theta^2 - 3\beta^2; \tag{25}$$

$$\Delta_1 = \begin{vmatrix} e & \beta & \gamma \\ f & \gamma & \theta \\ g & \theta & 3 \end{vmatrix} = 3e\gamma + f\theta\gamma + \beta g\theta - g\gamma^2 - e\theta^2 - 3\beta f; \tag{26}$$

$$\Delta_2 = \begin{vmatrix} \alpha & e & \gamma \\ \beta & f & \theta \\ \gamma & g & 3 \end{vmatrix} = 3\alpha f + \beta g\gamma + \gamma e\theta - f\gamma^2 - \alpha g\theta - 3\beta e; \tag{27}$$

$$\Delta_3 = \begin{vmatrix} \alpha & \beta & e \\ \beta & \gamma & f \\ \gamma & \theta & g \end{vmatrix} = \alpha\gamma g + \beta\theta e + \beta\gamma f - e\gamma^2 - \alpha\theta f - \beta^2 g. \tag{28}$$

This results in a Cramer system, also known as the "uniquely determined" system, on the basis of which the values of the nodes required to validate the mathematical model are determined.

The Cramer system is defined on the basis of relations (29), (30) and (31) as follows:

$$a^* = \frac{\Delta 1}{\Delta S} = \frac{3e\gamma + f\theta\gamma + \beta g\theta - g\gamma^2 - e\theta^2 - 3\beta f}{3\alpha\gamma + 2\beta\gamma\theta - \gamma^3 - \alpha\theta^2 - 3\beta^2} \quad (29)$$

$$b^* = \frac{\Delta 2}{\Delta S} = \frac{3\alpha f + \beta g\gamma + \gamma e\theta - f\gamma^2 - \alpha g\theta - 3\beta e}{3\alpha\gamma + 2\beta\gamma\theta - \gamma^3 - \alpha\theta^2 - 3\beta^2} \quad (30)$$

$$c^* = \frac{\Delta 3}{\Delta S} = \frac{\alpha\gamma g + \beta\theta e + \beta\gamma f - e\gamma^2 - \alpha\theta f - \beta^2 g}{3\alpha\gamma + 2\beta\gamma\theta - \gamma^3 - \alpha\theta^2 - 3\beta^2} \quad (31)$$

The Cramer type system is a linear system of n equations having n unknowns. It is called a Cramer system if the matrix determinant of the coefficients of the system obtained is non-zero (is non-zero). The Cramer type system, or single determined system, (compatible determined) **has only one solution.** The mathematical model for calculating the values of θ, g, γ, β, α, e and f is defined and presented in Table 7.

Table 7. Mathematical calculation model for obtaining the values of θ, g, γ, β, α, e and f.

	s	**y**	**s^2**	**s^3**	**s^4**	**s^2y**	**sy**
	s_1	y_1	s_1^2	s_1^3	s_1^4	$s_1^2y_1$	s_1y_1
	s_2	y_2	s_2^2	s_2^3	s_2^4	$s_2^2y_2$	s_2y_2
	s_3	y_3	s_3^2	s_3^3	s_3^4	$s_3^2y_3$	s_3y_3
$\sum_{k=1}^{3}$	$s_1+s_2+s_3$	$y_1+y_2+y_3$	$s_1^2+s_2^2+s_3^2$	$s_1^3+s_2^3+s_3^3$	$s_1^4+s_2^4+s_3^4$	$s_1^2y_1+s_2^2y_2+s_3^2y_3$	$s_1y_1+s_2y_2+s_3y_3$
	θ	**g**	**γ**	**β**	**α**	**e**	**f**

were,

s_1 = 30 km/h; s_2 = 50 km/h; s_3 = 130 km/h; y - noxa (CO_2). (32)

Therefore,

$$y \simeq h^* = h(v; a^*, b^*, c^*) \quad (33)$$

Observation: For s = s_0 (e.g. for speeds other than those used in the research such as: 10 km/h, 20 km/h, 30 km/h, 40 km/h, 50 km/h, 60 km/h, 70 km/h, 80 km/h), and so on, we got it:

$$y \simeq h^* = h(s_0; a^*, b^*, c^*) = a^* \cdot s_0^2 + b^* \cdot s_0 + c^* \quad (34)$$

After obtaining the values of θ, g, γ, β, α, e and f, according to the mathematical model in Table 6, substitute in the mathematical relations (29), (30) and (31) on the basis of which the validation of the mathematical model for the noxes "y" is reached.

2.3 The mathematical model for the determination of CO_2 emissions and its validation

The validation of the mathematical model for the average values of CO_2 (carbon dioxide) "y" emitted by the combustion engine of the Toyota RAV 4 hybrid, Euro 6 car is presented below.

For the validation of the mathematical model, the average values of chemical noxides in Table 5 will be considered.

Table 8 shows the mathematical calculation model for obtaining the values of θ, g, γ, β, α, e and f of CO_2 emissions (%) for the Toyota RAV 4 hybrid, Euro 6 car.

Table 8. Mathematical calculation model for obtaining values of θ, g, γ, β, α, e and f, of CO_2 emissions (%), for Toyota hybrid, Euro 6 car.

	s	**y**	**s^2**	**s^3**	**s^4**	**s^2y**	**sy**
	30	9,33	30^2	30^3	30^4	30^2 x 9,33	30 x 9,33
	50	11,6	50^2	50^3	50^4	50^2 x 11,6	50 x 11,6
	130	12,87	130^2	130^3	130^4	130^2 x 12,87	130 x 12,87
$\sum_{k=1}^{3}$	**210**	**33,8**	**20.300**	**2.349.000**	**292.670.000**	**254.900**	**2.533**
	θ	**g**	**γ**	**β**	**α**	**e**	**f**

Substituted into the mathematical relations (29), (30) and (31), the values are obtained:

$$\mathbf{a^*} = \frac{\Delta 1}{\Delta S} = \frac{3e\gamma + f\theta\gamma + \beta g\theta - g\gamma^2 - e\theta^2 - 3\beta f}{3\alpha\gamma + 2\beta\gamma\theta - \gamma^3 - \alpha\theta^2 - 3\beta^2} = \quad (35)$$

$$= \frac{3x254{,}9x20{,}3 + 2{,}533x0{,}210x20{,}3 + 2.349x0{,}0338x0{,}210 - 0{,}0338x20{,}3^2 - 254{,}9x0{,}210^2 - 3x2.349x2{,}533}{3x292.670x20{,}3 + 2x2.349x20{,}3x0{,}210 - 20{,}3^3 - 292.670x0{,}210^2 - 3x2.349^2} = \quad (36)$$

$$= \frac{-24.992}{25.600.000} = -0{,}00097625; \quad \mathbf{a^* = -0{,}00097625.} \quad (37)$$

$$\mathbf{b^*} = \frac{\Delta 2}{\Delta S} = \frac{3\alpha f + \beta g\gamma + \gamma e\theta - f\gamma^2 - \alpha g\theta - 3\beta e}{3\alpha\gamma + 2\beta\gamma\theta - \gamma^3 - \alpha\theta^2 - 3\beta^2} = \quad (38)$$

$$= \frac{3x292.670x2{,}533 + 2.349x33{,}8x20{,}3 + 20{,}3x254{,}9x0{,}210 - 2{,}533x20{,}3^2 - 292.670x0{,}0338x0{,}210 - 3x2.249x254{,}9}{3x292.670x20{,}3 + 2x2.349x20{,}3x0{,}210 - 20{,}3^3 - 292.670x0{,}210^2 - 3x2.349^2} = \quad (39)$$

$$= \frac{4.904.960}{25.600.000} = 0{,}1916; \quad \mathbf{b^* = 0{,}1916.} \quad (40)$$

$$\mathbf{c^*} = \frac{\Delta 3}{\Delta S} = \frac{\alpha\gamma g + \beta\theta e + \beta\gamma f - e\gamma^2 - \alpha\theta f - \beta^2 g}{3\alpha\gamma + 2\beta\gamma\theta - \gamma^3 - \alpha\theta^2 - 3\beta^2} = \quad (41)$$

$$= \frac{292.670x20{,}3x0{,}0338 + 2.349x0{,}210x254{,}9 + 2.349x20{,}3x2{,}533 - 254{,}9x20{,}3^2 - 292.670x0{,}210x2{,}533 - 2.349^2x0{,}0338}{3x292.670x20{,}3 + 2x2.349x20{,}3x0{,}210 - 20{,}3^3 - 292.670x0{,}210^2 - 3x2.349^2} = \quad (42)$$

$$= \frac{114.192.000}{25.600.000} = 4{,}46062; \quad \mathbf{c^* = 4{,}46062.} \quad (43)$$

The mathematical model obtained is:

$$\%CO_2 = a^* \cdot s^2 + b^* \cdot s + c^*; \quad \%CO_2 = 0{,}00097625 \cdot s^2 + 0{,}1916 \cdot s + 4{,}46062 \quad (44)$$

The graphical representation of the mathematical model is shown in Figure 7.

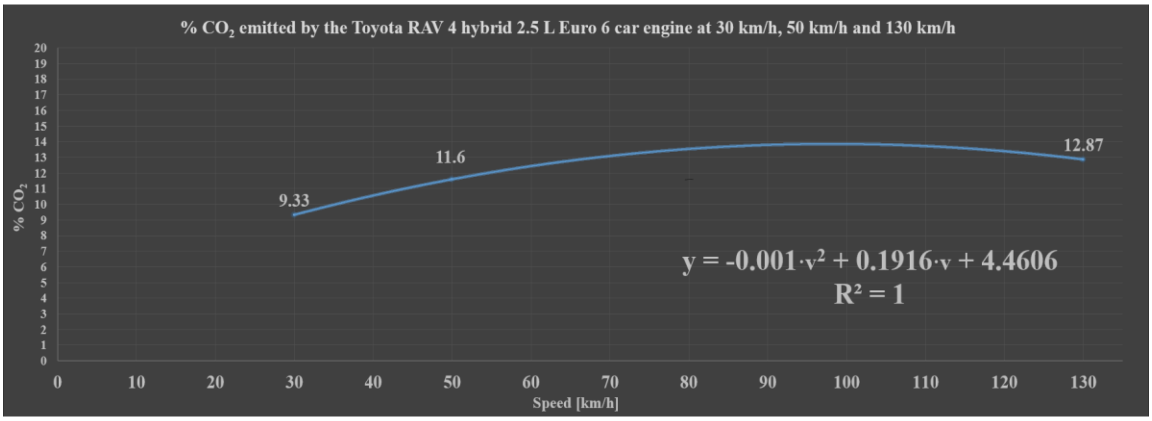

Figure 7. CO_2 (%) emitted by Toyota RAV 4 hybrid 2.5 liters, Euro 6 car at speed of 30 km/h, 50 km/h and 130 km/h.

Apply in relation (34) and obtain for speeds in the range [0-130] km/h, the CO_2 emission return values (%) corresponding to each speed in the range **y = $CO_{2\ \text{Toyota RAV 4 hybrid car}}$ = [0-12,87]%**, such:

a) For s_0 = 10 km/h, we got: y ≃ h* = h (-0,00097625; 0,1916; 4,46062) = -0,00097625x102 + 0,1916x10 + 4,46062 = 6,28%; (45)

b) For s_0 = 20 km/h, we got: y ≃ h* = h (-0,00097625; 0,1916; 4,46062 = -0,00097625x20^2 + 0,1916x20 + 4,46062 = 7,90%; (46)

c) For s_0 = 30 km/h, we got: y ≃ h* = h (-0,00097625; 0,1916; 4,46062) = -0,00097625x30^2 + 0,1916x30 + 4,46062 **= 9,33%;** (47)

d) For s_0 = 40 km/h, we got: y ≃ h* = h (-0,00097625; 0,1916; 4,46062) = -0,00097625x40^2 + 0,1916x40 + 4,46062 = 10,56%; (48)

e) For s_0 = 50 km/h, we got: y ≃ h* = h (-0,00097625; 0,1916; 4,46062) = -0,00097625x50^2 + 0,1916x50 + 4,46062 **= 11,6%;** (49)

f) For s_0 = 60 km/h, we got: y ≃ h* = h (-0,00097625; 0,1916; 4,46062) = -0,00097625x60^2 + 0,1916x60 + 4,46062 = 12,44%; (50)

g) For s_0 = 70 km/h, we got: y ≃ h* = h (-0,00097625; 0,1916; 4,46062) = -0,00097625x70^2 + 0,1916x70 + 4,46062 = 13,09%; (51)

h) For s_0 = 80 km/h, we got: y ≃ h* = h (-0,00097625; 0,1916; 4,46062) = -0,00097625x80^2 + 0,1916x80 + 4,46062 = 13,54%; (52)

i) For s_0 = 90 km/h, we got: y ≃ h* = h (-0,00097625; 0,1916; 4,46062) = -0,00097625x90^2 + 0,1916x90 + 4,46062 = 13,79%; (53)

j) For s_0 = 100 km/h, we got: y ≃ h* = h (-0,00097625; 0,1916; 4,46062) = -0,00097625x100^2 + 0,1916x100 + 4,46062 = 13,85%; (54)

k) For s_0 = 110 km/h, we got: y ≃ h* = h (-0,00097625; 0,1916; 4,46062) = -0,00097625x110^2 + 0,1916x110 + 4,46062 = 13,72%; (55)

l) For s_0 = 120 km/h, we got: y ≃ h* = h (-0,00097625; 0,1916; 4,46062) = -0,00097625x120^2 + 0,1916x120 + 4,46062 = 13,39%; (56)

m) For s_0 = 130 km/h, we got: y ≃ h* = h (-0,00097625; 0,1916; 4,46062) = -0,00097625x130^2 + 0,1916x130 + 4,46062 **= 12,87%.** (57)

For the speed range developed by the Toyota hybrid, 2.5 liters, Euro 6 car, the %CO_2 values are shown with a 10 km/h step in Figure 8.

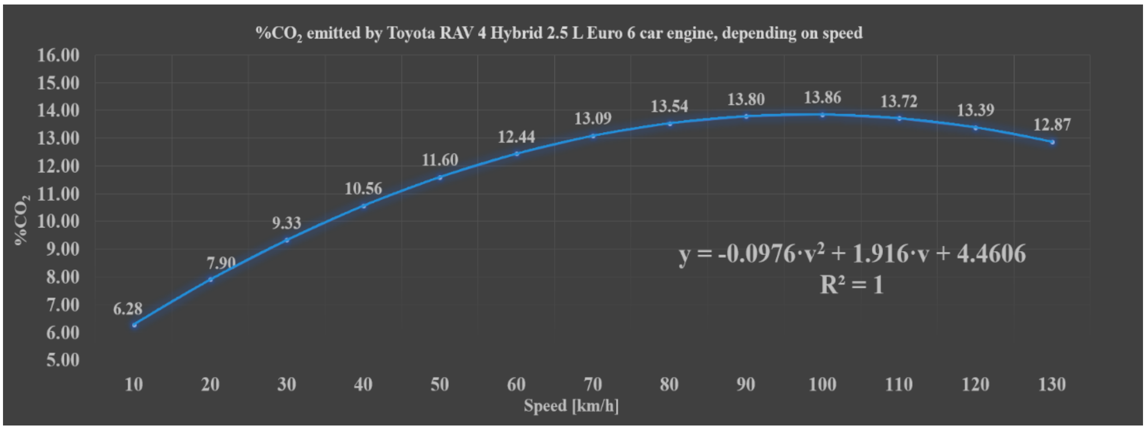

Figure 8. %CO_2 emitted by Toyota RAV 4 hibrid 2.5 liters, Euro 6 car, depending on speed.

The values are plotted as a parabola and the CO_2 values at different speeds of the Toyota hybrid car correspond to the proposed mathematical model, i.e: $y = h(s; a, b, c) = a\text{-}s^2 + b\text{-}s + c$.

3. Conclusions

By applying the method of least squares, we obtain values of the noxious "y", corresponding to the speed in the range [0-130] km/h i.e, $y = CO_{2\text{Toyota RAV 4 hybrid car [0-130] km/h}} = [0 – 12{,}87]$ %. It can be seen that the values of CO_2 (%) obtained by mathematical calculation for speeds of 30 km/h, 50 km/h and 130 km/h respectively are identical to the average values of CO_2 (%) obtained in real life by the experimental method at the same speeds (30 km/h = 9.33%; 50 km/h = 11.6%; 130 km/h = 12.87%). The mathematical model shows a high correlation stage ***(correlation coefficient - determination is 0.99)***. Consequently, the mathematical model applied on the average CO_2 values (%) for the Toyota RAV 4 hybrid car ***is valid***. Both the mathematical formulas and the calculations confirm the results returned by the measuring equipment for CO_2 (%) emitted by the car engine. The mathematical relationships (formulas) presented in the mathematical model are the basis for the operation of the measurement instrument (KANE 5+1 gas analyser). They form the basis of the hardware on which the KANE 5+1 gas analyser, used in the experimental research, operates.

In the European Union and Romanian legislation, there is no stipulated maximum reference value (Table 1) for chemical noxious emissions from motor vehicles with spark ignition and compression ignition engines. It is therefore we consider that the maximum CO_2 emissions of the car engine are normal emissions, specific to the Euro 6 pollution standard.

Further research directions and applications of mathematical modelling are as follows:

- Application and validation of the mathematical model also on other types of experimentally obtained chemical noxides (carbon monoxide - CO, hydrocarbons - HmCn; nitrogen oxides - NOx);
- Application and validation of the mathematical model on noise noises and noises produced in the passenger compartment of motor vehicles in order to determine the vibrational comfort perceived by users or travellers during the provision of transport services by road vehicles;
- Application and validation of the mathematical model on noise and noises produced in road vehicle traffic.

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