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Mortality Laws Usage in Forecasting Life Expectancy – A Macroeconomic Concern

Francesca Dana Andreescu

Doctoral School of Finance, Academy of Economic Studies, Bucharest Romania

E-mail: andreescu.francesca@yahoo.com

Abstract. The increasing number of survivors at advanced ages represents a real economic challenge, with high impact over the pension system and over the principal macroeconomic indicators, even if there are studies which emphasise the positive impact of an increasing life expectancy over economic development. In this case, an accurate estimation is essential in improving predictions, considering that a continuous aging of the population is expected in the coming decades. In this sense, parameterized mortality models are useful tools in demographic and actuarial projections. Therefore, this study aims to present a forecast for life expectancy, after estimating mortality rates and determining mortality table.
Keywords: *Mortality laws, mortality rates, mortality table, life expectancy.*

1. Introduction

Over the years, there have been several attempts to draw mathematical formulas in order to highlight properly the dependence between mortality rates and age, these having a wide application in population projection by facilitating the actuarial process of building mortality tables.

As it has been observed, global mortality is in a continuous decline since the beginning of century, while life expectancy increases. These improvements affect the calculation of pensions provisions and have impact over the pension system, representing a real challenge for actuaries and demographers who model longevity.

There are also studies that investigate the causality between life expectancy and economic growth, revealing the fact that life expectancy improvement leads to reducing population growth, but also to sustained income growth, being associated, in some cases, with a high income per capita.

On the other hand, there are several studies which rise the concern about the fact that longevity may lead to a slowing of economic growth due to an increase in social spending on elders, with negative impact over productivity growth and capital accumulation.

In most developed countries, fertility has already reached a very low level and the chances for radical changes are small. Thus, mortality, especially at advanced ages represents the main concern regarding future population fluctuations.

Within actuarial mathematics, are studied models that can be used in order to determine the factors that influence survival and death of an established population. Biometrics is the science that deals with quantitative measurement in such cases, through biometric functions.

2. Literature Review

One of the most important approaches of the subject belongs to the English actuary Benjamin Gompertz [4], which describes the mathematical dependence between mortality and age. In his works, Gompertz highlights that, apart from the mortality that increases exponentially with age, there is also a mortality age-independent component, called, 35 years later, the Makeham parameter [6].

Later, numerous researchers attempt to modify Gompertz's law by using logistic equations. Recently, Bongaarts [2] develops the method based on the historical trend of the Gompertz-Makeham parameters, suggesting the use of logistic formulas for mortality predictions. However, despite the usefulness, the parametric approach of mortality projections has limitations as the dependence on a specific formula, which leads to a rigid response to possible fluctuations.

Nowadays, Lee-Carter's method is one of the most widespread and used projection methods in this sense. Its model is not based on a parameterized formula and allows a compact description of a set of mortality data without excessive loss of information.

A limitation of this method is related to the assumption according to which the historical evolution of mortality for any group of age is given by a single factor, proving the simplicity of this approach. Therefore, this study aims to explore the use of Lee-Carter method in forecasting Romanian population mortality rate between 2023 and 2030.

3. Forecasting Life Expectancy through Lee-Carter Method

Lee-Carter method is used for long-term estimations of mortality level, being based on a sequence of time series specific methods and on the analysis of mortality distribution.

The method states that the logarithm of the death rates $m_{x,t}$ series captures each age specific death rates as the sum of a time-independent component and a component that represents the product between a time varying parameter, which reflect the general level of mortality, and an age-specific parameter, which captures mortality variation speed, at each age, while the general mortality level changes, as seen in equation (1).

$$f_{x,y} = \ln(m_{x,t}) = a_x + b_x k_t + \epsilon_{x,t} \tag{1}$$

The data is divided into age intervals, so the interval with the lower limit x is called *age x interval*. Therefore,

- $m_{x,t}$ is the mortality rate specific to the x interval,
- k_t represents the mortality index in year t which can capture 80%-90% from the historical mortality trend,
- a_x is the mortality specific to the interval average age,
- b_x shows the mortality deviation caused by the changes in the morality index, reported to total deviation,
- $\epsilon_{x,t}$ is the error which follows a normal distribution N(μ,σ).

In fact, the model represents a simultaneous equations system, depending on time and age. The mortality rates for the defined r age intervals, create an equation system with 2r+n unknown coefficients corresponding to the r values of a_x and b_x parameters, and to the n values associated to k_t.

According to those stated above, Lee-Carter parameters for each age in case of Romanian total population between 1990-2022 can be estimated, based on Eurostat age specific death rates. Figure 1 emphasis the evolution of life expectancy at birth age during the analyzed period.

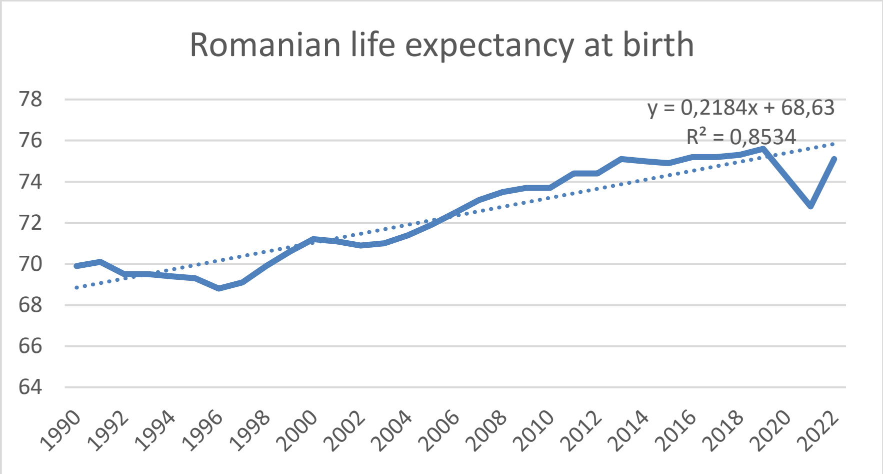

Figure 1. Life expectancy at birth for Romanian total population between 1990-2022

As it might be seen, the life expectancy in Romania has an increasing tendency during the period subject to this study, with a drop in 2020 and 2021.

In order to determine the Lee-Carter parameters for Romanian population and to achieve the goal of forecasting life expectancy, it is necessary, first of all, to determine the logarithm values of the death rates for each age. The death rate for an interval average age is calculated based on equation (2).

$$a_x = \frac{\sum_{t=1}^{n} ln\, m_{x,t}}{n} \tag{2}$$

At first glance, there is no unique solution for the system, but according to the above conditions, a_x is, in fact, the arithmetic mean of the mortality rate logarithm over time, and its exponential value is the general shape of the mortality in Romania, with a decreasing pattern until 11 years old and an increasing pattern staring with 12 years old age group.

The next step is to center the death rates in matrix M, which means extracting the vector a from its columns by applying equation (3). Further, to the resulting matrix is applied singular value decomposition (SVD).

$$M^* = M - a = bk \tag{3}$$

Lee-Carter model cannot be described using a simple regression because the variables from the right side of the equation are unobservable variables, therefore least squares method can be used. For structuring the matrix decomposition, in order to assure the solution uniqueness without restricting the generality, the following conditions will be applied:

$$\sum_{x=1}^{\omega} b_x = 1 \tag{4}$$

$$\sum_{t=1}^{n} k_t = 0 \tag{5}$$

After applying, in Eviews, the singular value decomposition of matrix M^* into U and V matrices and vector S, both b_x and k_t are derived as it follows:

$$k_t = s_1(u_{1,1}, u_{2,1}, \dots, u_{t,1})$$
$$b_x = (v_{1,1}, v_{1,2}, \dots, v_{1,x})$$

(6)

Even if Lee-Carter did not analyze the possibility of strong variations over time of b_x, in 1994, Kannisto discovers that the mortality rate decreased more and more rapidly, in the last decades, for older ages, a fact also demonstrated by Horiuchi and Wilmoth in 1995, as an exception to the historical pattern.

Further, in order to forecast Romanian mortality rates between 2023-2030, after analyzing k_t seasonality (applying Augmented-Dicky-Fuller test) and correlogram, the index is modelled using an ARMA (1,4) process (Table 1), which proves to be stable.

Table 1. Seasonally autoregressive process for general mortality index k_t

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.952241	0.050059	19.02226	0.0000
MA(4)	0.514914	0.213057	2.416786	0.0219
SIGMASQ	0.513581	0.102860	4.993000	0.0000
R-squared	0.795481	Mean dependent var		-2.941827
Adjusted R-squared	0.781846	S.D. dependent var		1.609236
S.E. of regression	0.751624	Akaike info criterion		2.483983
Sum squared resid	16.94818	Schwarz criterion		2.620029
Log likelihood	-37.98571	Hannan-Quinn criter.		2.529758
Durbin-Watson stat	2.310442			

Table 2. General mortality index k_t forecast between 2023-2030

	2023	2024	2025	2026	2027	2028	2029	2030
k_{t+n}forecast	-2.1043	-2.0983	-2.0708	-2.0782	-1.9790	-1.8845	-1.7945	-1.7088

The forecasted values for the general mortality index are presented in Table 2 and are used in order to predict mortality rates between 2023-2030, based on the following formula:

$$m_{x,t+n} = m_{x,t} \exp\,(b_x(k_{t+nforecast} - k_t)$$

(7)

In order to construct future mortality tables, we taken into account, for each age, the followings: death rates m_x, death probability (q_i), number of survivors (l_x), number of deaths (d_x), number of years lived in interval (L_x), total number of years lived until the starting age of the interval (T_x) and life expectancy (e_x).

Starting from the general mortality index, we determine the death rates forecast, following which the probabilities of death can be calculated, based on formula (8).

$$q_i = \frac{m_i}{1 + (1 - a_i)m_i}$$

(8)

According to equation (8), a_i is the average of the fractions of the interval lived by the individuals who died during that certain interval. For most of the age intervals, a_i is considered to be 0.5, meaning that, in average, the individuals who die in that interval reach to survive half of the time horizon. There are certain studies which reveal the fact that a_i register a much lower value for individuals that do not survive until the end of the first year of life, namely 0.09, while between 1 year and 9 years old is slightly smaller than 0.5.

Further, the estimated number of deaths during the interval is calculated starting from the estimated number of survivors and the probability of death, according to the formula (9).

$$d_i = l_i q_i \tag{9}$$

All the other mortality table elements are determined as it follows:

$$L_x = a_x l_x + (1 - a_x) l_{x+1}, \qquad x = 0$$

$$L_x = n_x \frac{l_x + l_{x+1}}{2}, \qquad x > 0$$

$$T_i = \sum_{k=i}^{\omega} L_k$$

$$e_i = \frac{T_i}{l_i} \tag{10}$$

Table 3. Life expectancy at birth forecast between 2023-2030

	2023	2024	2025	2026	2027	2028	2029	2030
Life exp. at birth	73.92	73.93	73.94	73.93	73.98	74.02	74.06	74.09

Life expectancy estimated using Lee-Carter model (Table 3) shows insignificant increase during the forecasting period, probably due to the fact that the sustained mortality rates decrease over the years before COVID-19 cannot compensate entirely the huge increase in 2020 and 2021. Therefore, it must be highlighted the fact that an increasing trend of life expectancy is not the only criteria that a good forecasting model should satisfy.

4. Conclusions

Life expectancy at birth registered a slight increase during the passing of centuries, while in the demographic and actuarial sciences there were several attempts to discover a suitable model for determining mortality. Traditionally, a parametric curve, similar to those proposed by Gompertz was used to calculate the annual death rates for a better life expectancy forecast, but in recent years, numerous approaches have appeared which aimed to predict mortality using a stochastic model.

However, Lee-Carter model has become one of the most well-known models, being applied in several countries and appearing in numerous studies on mortality forecasting.

The application of this model on the mortality tables of Romanian population between 1990-2022 aimed to determine the future evolution of death rates and life expectancy. The calculation highlights the fact that the first age intervals, corresponding to the infant population and early childhood, are characterized by a significantly higher mortality than in case of the other age groups.

The specific death rates for the period 2022-2030 shows insignificant variations at age range levels and are used to construct the mortality tables, obtaining a life expectancy with a small increasing tendency, probably due to the heavy impact of COVID-19 period.

Rapid life expectancy increases may lead to a sustainable development, taking into account that it is one of the most relevant indicators regarding population health and a synthetic indicator that may be use in order to assess economic development of a region, while its increasing is associated with educational level improvement, minimized unemployment and life conditions improvement. Still, there are evidences regarding the negative effect of improved life expectancy over economic growth.

However, these findings are not empirically supported in all cases, therefore there is no consensus regarding the way life expectancy may influence the development of an economy, but only a general acceptance regarding the existence of a relationship between those two variables. Therefore, future studies may assess the impact of life expectancy evolution over different economic growth indicators, in order to analyse if the relationship is stable over time and persistent from country to country.

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