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Vibration analysis of shaft-bearing assembly for centrifugal fan

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Abstract. The complete solution of the problem of a free vibration system needs the determination of all the modal frequencies and of correspondent mode of vibration. Many times in practice it is necessary to know only some modal frequencies, sometimes only the fundamental one. The classical method of the vibration problem solution is to write one or more movement equations, using the second Newton's law. For the systems with distributed parameters, the result is a number of differential equations with partial derivatives. The exact solution of the equations is possible only for a relative low number of cases; for the majority of the problems the approximate solving methods must be used. The paper presents comparative studies for the shaft-bearing assembly of the centrifugal fans, regarding the vibration behavior during the service. Two type of modeling has been realized: the first one was an analytical simulation for the ideal shaft, using Mathcad software. The second modeling was a numerical simulation for the real geometrical shaft, taking into account the presence of the bearings, using ANSYS software with all the possibilities of optimization involved. A comparison of the theoretical results between the two modeling has been realized.
Keywords: *fan, frequencies, mathcad, optimization.*

1. Introduction

The complete solution of the problem of a free vibration system needs the determination of all the proper frequencies and of correspondent vibration ways. Many times in practice it is necessary to know only some proper frequencies, sometimes – only one. Usually, the lower frequency is practically the most important. In most cases, the exact knowledge of the vibration way has a minor importance and that allows to elaborate new methods founding the frequencies of a low harmonics number [1], [2], [3].

The classical method of the vibration problem solution is to write one or more movement equations, using the second Newton's law [4]. For the systems with distributed parameters, the result is a number of differential equations with partial derivatives. The exact solution of the equations is possible only for a relative low number of cases; for the majority of the problems the approximate solving methods must be used [5].

Various methods have been used in analyzing the rotary systems, while standard transfer matrix method is given in many handbooks [6], [7]. Analytical results are being generated to demonstrate the need for and the advantage of transfer matrix method in modelling procedures. A quantitative comparison is made between the Finite Elements Method and Transfer Matrix Method, applied to free vibration analysis of rotor systems [8].

A mathematical model governing the transverse vibration of rotor shaft is determined using analytical methods [4]. From the mathematical model, natural frequencies and mode shape of the rotor system is determined. Further, numerical simulation is performed in ANSYS for modal analysis for such system. Unbalance effect is incorporated numerically by performing harmonic analysis of the rotor system in ANSYS [9].

2. Theoretical model

In the theoretical calculus of the proper bending frequencies of the fan shaft it will be used the classical differential equation for the transversal vibrations of the beam [1]:

$$-\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}\right) = \gamma A(g)^{-1}\frac{\partial^2 y}{\partial t^2} \qquad (1)$$

This equation is available with the following hypotheses:

- the material is homogenous, izothrop and perfectly elastic;
- the beam is straight and with the cross section constant for all the length;
- the deformations of the beam are low and its length is much greater, comparative with the dimensions of the cross section.

If the product *EI* is constant, the solution of the equation is:

$$y = X(x)[cos(\omega_n t + \theta)] \qquad (2)$$

If $k^4 = \omega_n^2\gamma A(EIg)^{-1}$ and dividing equation (1) by $cos(\omega_n t + \theta)$, it obtains:

$$\frac{d^4X}{dx^4} = k^4X \qquad (3),$$

where *X* is a function for which the fourth derivative is equal with the constant multiplied by the function itself.

The solution of the equation (3) is given by a sum of linear independent functions as:

$$X = A_1\, sin\, k\, x + A_2\, cos\, k\, x + A_3 sh\ kx + A_4 ch\ kx \qquad (4)$$

For the beams having different leaning conditions, the constants A_1, A_2, A_3 and A_4 are established from the limit conditions. It is convenient for founding solutions that the equation (4) must be written like (5), in which two constants are nulls, for each usual limit conditions:

$$X = A(coskx + coshkx) + B(coskx - coshkx) + C(sinkx + sinhkx) + D(sinkx - sinhkx) \qquad (5)$$

Applying the limit conditions, the following relations are used:

- the deformation is proportional with *X* and it is null on a stern support;
- the rotation is proportional with *X*' and it is null at a fixed end;
- the bending moment is proportional with *X*'' and it is null at a free or articulated end;
- the cutting force is proportional with *X*''' and it is null at a free end.

For the usual limit conditions, two constants are zero, and it obtains two equations with two constants. Those can be combined and we obtain an equation that contains the frequency as unknown. Using the frequency, one of the constants can be couch – one function other. Thus, a constant remains always undetermined; it can be evaluated only in the case when the vibration amplitude is known.

3. Analytical simulation for the ideal shaft

In the case of the continuous beams on many supports, for founding the modal frequencies, the section between each pair of supports is considered as a separate beam, with the origin in the left section support. The deformation equation is applied for each interval. There is such an equation for each section and the limit conditions are:

- At the ends of the beam we applied the usual limit conditions, function the type of the support.
- On each intermediate support, the deformation is null. Because the beam is continuous, at the left and at the right, in the neighborhood of the support, the rotation and the moment are the same.

A general view of the fan shaft is presented in Fig. 1, while the main components of the shaft bearings are shown in Fig. 2.

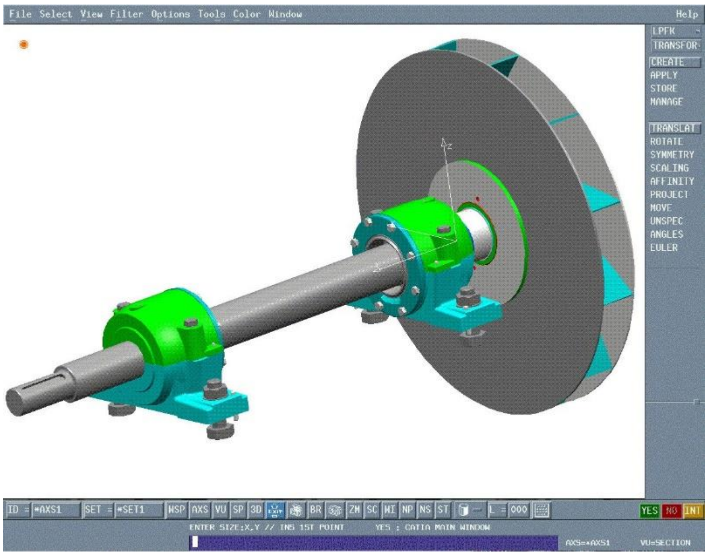


Figure 1. CAD model of the offset rotor shaft of centrifugal fan

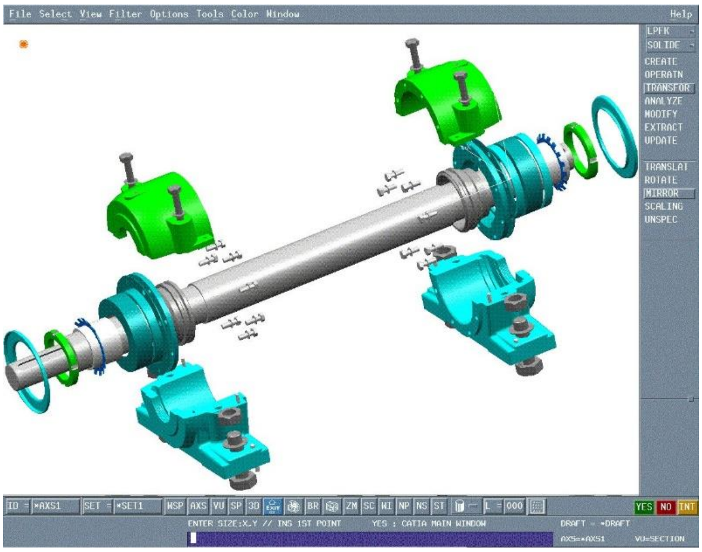


Figure 2. Details of the shaft bearings of centrifugal fan

The main hypotheses for the analytical modeling of the fan shaft are:

- The shaft is modeled like a continuous and homogenous beam, with a constant diameter;
- The equivalent beam is a continuous beam which is staying on two pinned supports (O_2 and O_4) and two free ends (O_1 and O_3) (Fig. 3);
- There are determined only the modal bending frequencies.

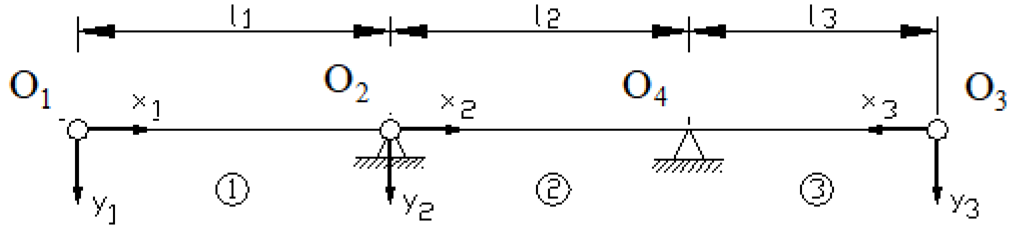


Figure 3. The simplified analytical model of the fan shaft

The geometrical dimensions of the shaft are: the medium diameter - 30 mm; the equivalent lengths: l_1 = 130 mm; l_2 = 200 mm; l_3 = 130 mm. The shaft material is OLC45, with the following mechanical characteristics: the elasticity modulus E = $2.1 \cdot 10^5$ MPa; the density ρ = 7800 kg/m^3. The deformation equations for the three regions are:

Region 1:

$$X_1 = A_1(\cos kx + \cosh kx) + B_1(\cos kx - \cosh kx) + C_1(\sin kx + \sinh kx) + \\ + D_1(\sin kx - \sinh kx) \tag{6}$$

Region 2:

$$X_2 = A_2(\cos kx + \cosh kx) + B_2(\cos kx - \cosh kx) + C_2(\sin kx + \sinh kx) + \\ + D_2(\sin kx - \sinh kx) \tag{7}$$

Region 3:

$$X_3 = A_3(\cos kx + \cosh kx) + B_3(\cos kx - \cosh kx) + C_3(\sin kx + \sinh kx) + \\ + D_3(\sin kx - \sinh kx) \tag{8}$$

The limit conditions imposed are:

At point O_1: $\begin{cases} X_1''(0) = 0 \\ X_1'''(0) = 0 \end{cases}$ At point O_2: $\begin{cases} X_1(l_1) = X_2(0) = 0 \\ X_1'(l_1) = X_2'(0) \\ X_1''(l_1) = X_2''(0) \end{cases}$

At point O_3: $\begin{cases} X_3''(0) = 0 \\ X_3'''(0) = 0 \end{cases}$ At point O_4: $\begin{cases} X_2(l_2) = X_3(-l_3) = 0 \\ X_2'(l_2) = X_3'(-l_3) \\ X_2''(l_2) = X_3''(-l_3) \end{cases}$

The equivalent equations system given by the limit conditions is a homogenous system with 12 equations with 12 unknowns (the coefficients $A_1, A_2, \ldots, D_3$) like:

$$M \cdot A = O \qquad (9),$$

where M is the coefficient matrix, A is the unknowns vector and O is the null vector corresponding to the free term.

The general equation (the case $l_1 \neq l_2 \neq l_3$) for founding the modal frequencies of the shaft is obtained by developing the matrix determinant M, using the symbolic facilities of the Mathcad software. The solution of the equation is obtained only numerical.

For the peculiar case when $l_1 = l_2 = l_3 = l$ (the case is analyzed in ref. [4], but for others supports types: fixed support – pinned support or pinned support – pinned support), the general equation for founding the modal frequencies has a simple form, but also without an analytical solution:

$$3\cosh x\cos^3 x + 8\cos^2 x\sin x\sinh x\cosh^2 x + 2\sinh x\sin x + 6\cos x\cosh x\sin x\sinh x -$$
$$-3\cos x\cosh^3 x + \cos^2 x - \cosh^2 x = 0 \quad (10),$$

where $x = kl$.

The peculiar solutions of this equation are corresponding to the first five mode of vibrations (Table 1).

Table 1. Peculiar values of the characteristic parameter $x = kl$

Mode of vibration	Parameter $x = kl$
1	1.648
2	4.706
3	6.707
4	7.854
5	9.849

Knowing the constant parameter values $x = kl$, the pulsation and the modal frequencies are obtained using the relations:

$$\omega_n = k_n^2\left(\frac{EIg}{\gamma A}\right)^{1/2} \quad ; \quad f_n = \omega_n(2\pi)^{-1} \quad (11),$$

where k_n is calculated for each mode of vibration, function of the corresponding parameters x and l.

For the general case of the fan shaft ($l_1 \neq l_2 \neq l_3$), the numerical solution obtained is presented in Table 2.

Table 2. Modal frequencies of the fan shaft (approximate analytical model)

Mode of vibration	Parameter k_n, mm^{-1}	Modal frequency f_n, Hz
1	0.00985	601
2	0.01205	899
3	0.02025	2540
4	0.03068	5828
5	0.03333	6899

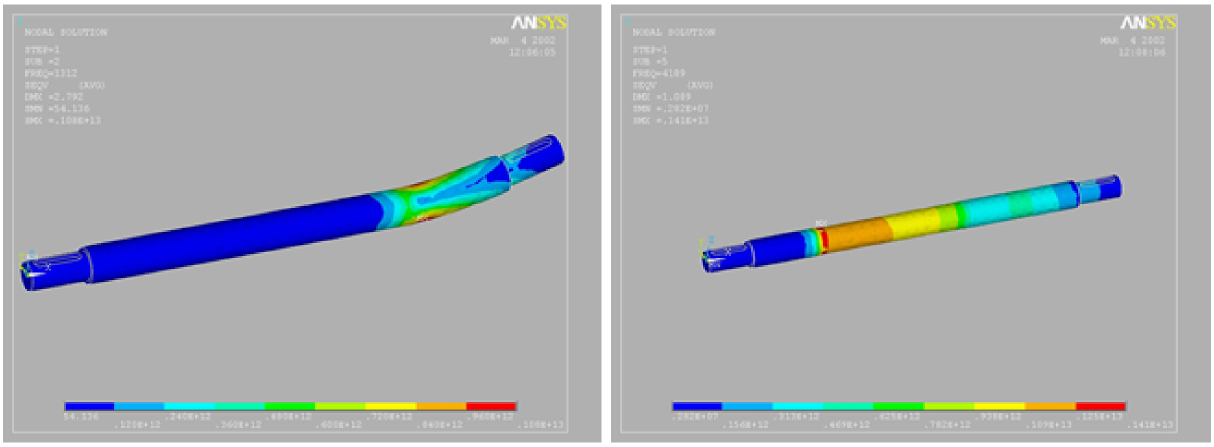
4. Numerical simulation for the real geometrical shaft

The numerical solution obtained with the aid of the finite elements software for the case of the shaft (existent in the shaft – bearings assembly) is presented in Figure 4 a and b.

Also, there are presented the results for the case of whole assembly shaft – rotor in Figure 5 a and b. The numerical values of the modal frequencies for these two real studied cases (shaft – bearing assembly and shaft – rotor assembly) are presented in the Table 3.

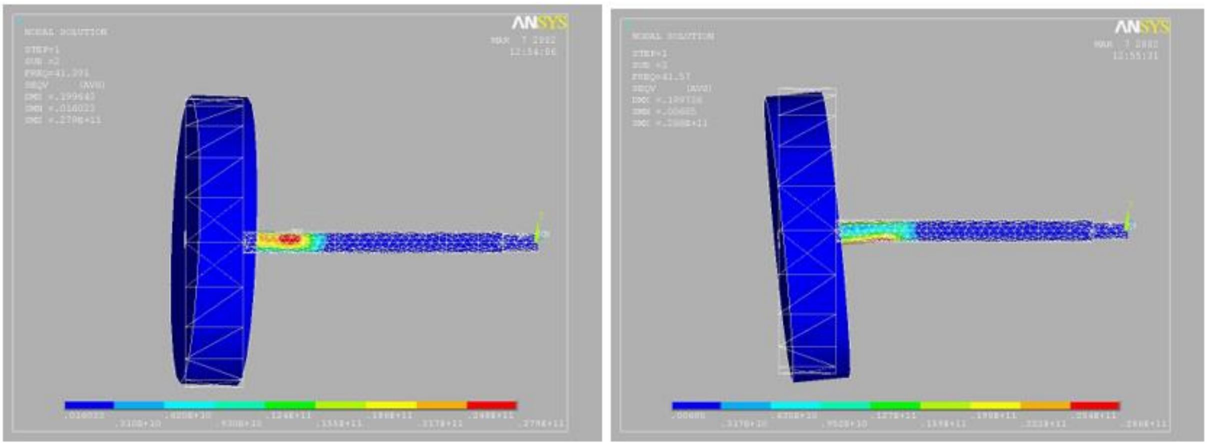
Table 3. Numerical values of the modal frequencies for real cases

Mode of vibration	Modal frequency f_n, Hz (shaft – bearings assembly)	Modal frequency f_n, Hz (shaft – rotor assembly)
1	1311	37
2	1312	41
3	3620	41
4	3667	239
5	4189	240



a. Modal frequency f_n = 1312 Hz   b. Modal frequency f_n = 4189 Hz

Figure 4. Mode of vibration for shaft – bearings assembly



a. Modal frequency f_n = 41.39 Hz   b. Modal frequency f_n = 41.57 Hz

Figure 5. Mode of vibration for shaft – rotor assembly

5. Conclusions

1. The analytical model proposed permits to establish the general equation, which determine the modal frequencies for a double pinned beam with free ends.
2. The differences existent between the real numerical model and the analytical simplified model for the shaft-bearings assembly are justified by:

- For the analytical case, only the modal bending frequencies are considered; in the numerical case, the modal frequencies result by composing the bending and torsion movement.
- The analytical model corresponds to a constant cross section beam, while the real shaft is characterized by diametrical variation.
- The real shaft is also characterized by the existence of two bearings, which have a significant influence on the results.

3. There are great differences from the point of view of the modal frequencies between the real model of the shaft-bearings assembly and the real model of the entire fan (shaft-rotor assembly). These differences can be explained by the major geometrical and inertial influence of the rotor on the behavior of the entire fan.

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