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Elastic linear calculation of a crane

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Abstract. This paper proposes a theoretical model for solving an indeterminate static system of straight crane bars. Starting from the basic model, the algorithm involves drawing bending moment diagrams for the considered model, writing the coefficients of influence and then the matrix equation - the system of equations, with the help of which the basis of which the unknowns considered are determined.
Keywords: *indeterminate static system, coefficients of influence, Veresceaghin procedure.*

Introduction

The theoretical model presented in the paper represents an algorithm for solving plane frames, statically indeterminate. The method used is based on the linear elastic calculation. Determining the unknowns involves writing the system of equations with the corresponding influence coefficients. To write them, the areas described by the momentary equations for real-force loading must be known, as well as those obtained by replacing the unknown data in the basic system with unit loads. In determining the coefficients of influence, the Veresceaghin procedure is used.

The theoretical model

The proposed theoretical model is a five-bar planar system with embedding, figure.1. In reality, it is the situation of a crane used to deserve certain economic activities

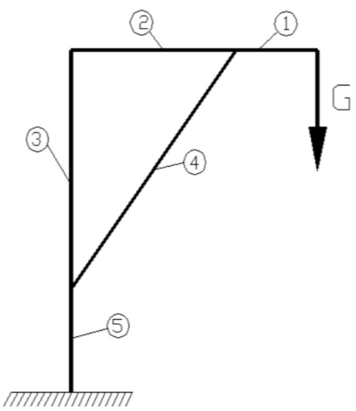

Figure 1. Theoretical loading model

Straight bars 1, 2, …5 have lengths l_1, l_2, l_3, l_4 and are characterized by axial moments of inertia I_1, I_2, I_3, I_4 and I_5.
Starting from the chosen basic system, figure 2,

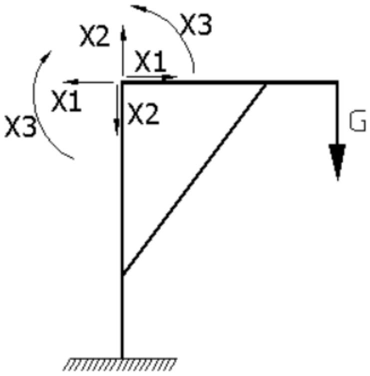

Figure 2. The proposed basic system

the associated system of equations is;

$$\begin{cases} \delta_{11}X_1 + \delta_{12}X_2 + \delta_{13}X_3 = -\Delta_{01} \\ \delta_{21}X_1 + \delta_{22}X_2 + \delta_{23}X_3 = -\Delta_{02} \\ \delta_{31}X_1 + \delta_{32}X_2 + \delta_{33}X_3 = -\Delta_{03} \end{cases} \quad (1)$$

where:

$$\left.\begin{matrix} \delta_{11}, \delta_{12}, \delta_{13} \\ \delta_{21}, \delta_{22}, \delta_{23} \\ \delta_{31}, \delta_{32}, \delta_{33} \\ \Delta_{01}, \Delta_{02}\ \Delta_{03} \end{matrix}\right\} \text{ influence coefficients and } \left.\begin{matrix} X_1 \\ X_2 \\ X_3 \end{matrix}\right\} \text{ unknown to the system.}$$

Trapezoid center of gravity

In order to determine the coefficients of influence, it is necessary to know the moment quotas, measured in the moment diagrams for unit loads, next to the centers of gravity of the areas of the moment diagrams obtained for the load with real force. The position of the center of gravity, figure 3 is given by relations 2:

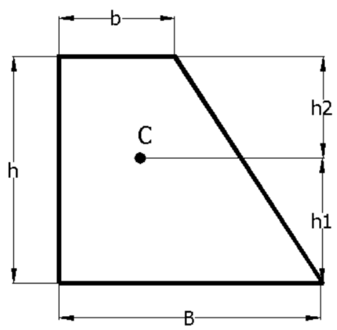

Figure 3. Position of center of gravity of the trapezoid

$$h1 = \frac{h}{3} \cdot \frac{2b + B}{B + b}$$

(2)

$$h2 = \frac{h}{3} \cdot \frac{b + 2B}{B + b}$$

Bending moment diagrams

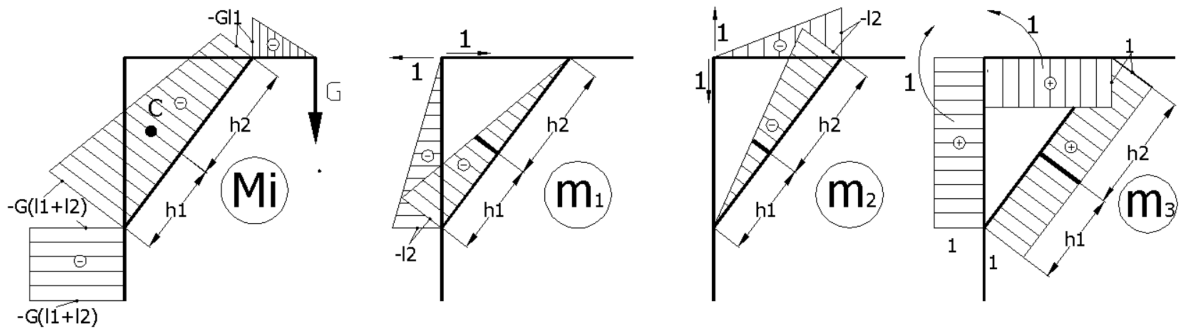

Figure 4. Bending moment diagrams, actual load and unit loads

Based on the obtained diagrams, figure 4, the relations were written

$$A_{tr} = -\frac{Q(2l_1 + l_2)l_4}{2} \qquad (3)$$

$$h1 = \frac{h}{3} \cdot \frac{3l_1 + l_2}{2l_1 + l_2}$$

$$h2 = \frac{h}{3} \cdot \frac{3l_1 + 2l_2}{2l_1 + l_2}$$

(4)

Bending moment quotas Y_1 and Y_2, measured in the moment diagrams obtained for the unit load are shown in the figure below, figure 5.

Figure 5. Current odds

Their values are calculated using relationships

$$\frac{Y1}{-l2}=\frac{h2}{l4}\Rightarrow Y1=-\frac{l2}{l4}\cdot h2$$

$$\frac{Y2}{-l2}=\frac{h1}{l4}\Rightarrow Y2=-\frac{l2}{l4}\cdot h1 \tag{5}$$

Coefficients of influence
In the following we will write the coefficients of influence like this:

$$\Delta01=\frac{Y1\cdot A_{tr}}{EI_4}$$

$$\Delta02=\frac{Y2\cdot A_{tr}}{EI_4} \tag{6}$$

$$\Delta03=\frac{A_{tr}}{EI_4}$$

$$\delta_{11}=\frac{-\frac{l_2\cdot l_4}{2}\cdot-\frac{l_2}{3}}{EI_4}+\frac{-\frac{l_2^2}{2}\cdot\left(-\frac{l_2}{3}\right)}{EI_3}\qquad \delta_{11}=\frac{l_2^2\cdot l_4}{6EI_4}+\frac{l_2^3}{6EI_3} \tag{7}$$

$$\delta_{12}=\frac{-\frac{l_2\cdot l_4}{2}\left(-\frac{l_2}{3}\right)}{EI_4}\qquad \delta_{12}=\frac{l_2^2\cdot l_4}{6EI_4}=\delta_{21} \tag{8}$$

$$\delta_{13}=\frac{-\frac{l_2\cdot l_4}{2}\cdot1}{EI_4}+\frac{-\frac{l_2}{2}\cdot1}{EI_3}\qquad \delta_{13}=-\frac{l_2\cdot l_4}{2EI_4}-\frac{l_2}{2EI_3}=\delta_{31} \tag{9}$$

$$\delta_{22}=\frac{-\frac{l_2^2}{2}\left(-\frac{2l_2}{3}\right)}{EI_2}+\frac{-\frac{l_2l_4}{2}\cdot\left(\frac{-2l_2}{3}\right)}{EI_4}\qquad \delta_{22}=\frac{l_2^3}{EI_2}+\frac{l_2^2l_4}{EI_4} \tag{10}$$

$$\delta_{23}=\frac{-\frac{l_2^2}{2}\cdot1}{EI_2}+\frac{-\frac{l_2l_4}{2}\cdot1}{EI_4}\qquad \delta_{23}=-\frac{l_2^2}{EI_2}-\frac{l_2l_4}{EI_4}=\delta_{32} \tag{11}$$

$$\delta_{33}=\frac{l_2}{EI_3}+\frac{l_2}{EI_2}+\frac{l_4}{EI_4}\qquad \delta_{33}=\frac{l_2}{EI_3}+\frac{l_2}{EI_2}+\frac{l_4}{EI_4} \tag{12}$$

Writing the system given in relation 1 in the form of a matrix, thus

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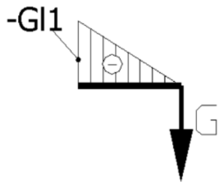
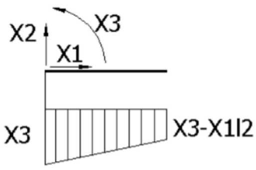
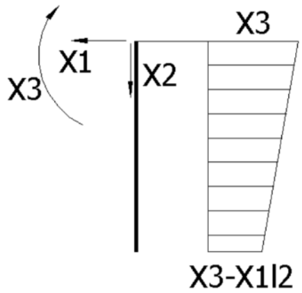
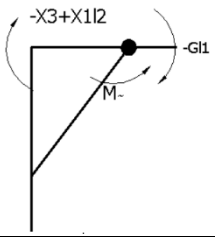
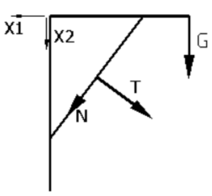
$$[\delta]\cdot\begin{bmatrix}X1\\X2\\X3\end{bmatrix}=-\begin{bmatrix}\Delta 01\\\Delta 02\\\Delta 03\end{bmatrix} \tag{13}$$

where:

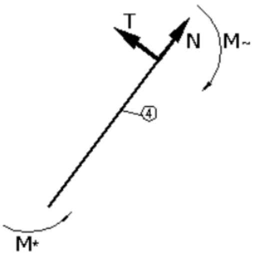
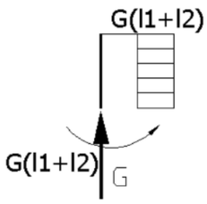
$[\delta]=-\begin{bmatrix}\delta_{11}&\delta_{12}&\delta_{13}\\\delta_{21}&\delta_{22}&\delta_{23}\\\delta_{31}&\delta_{32}&\delta_{33}\end{bmatrix}$ - the matrix of the influence coefficients is obtained unknowns $X1, X2, X3$.

Knowing $X1, X2, X3$ the bending moments are written on each bar:

<table>
<tr><td colspan="2">Bar 1
</td><td colspan="2">Bar 2
</td><td>Bar 3
</td></tr>
<tr><td>Bar 4

moments</td><td colspan="2"></td><td colspan="2">$\tilde{M}+X3-X1l_2-Gl_1=0$
$\tilde{M}=-X3+X1l_2+Gl_1$</td></tr>
<tr><td>forces</td><td colspan="2"></td><td colspan="2">$X1+N\cos\alpha-T\sin\alpha=0$
$X2+G+N\sin\alpha-T\cos\alpha=0$

$N\cos\alpha-T\sin\alpha=-X1\Rightarrow N=-X1\cos\alpha-(X2+G)\sin\alpha$
$N\sin\alpha-T\cos\alpha=-X2-G\Rightarrow T=X1\sin\alpha-(X2+G)\cos\alpha$</td></tr>
</table>

		$M^* = \widetilde{M} + Tl_4$
Bar 5		

Conclusions
The elastic linear calculation applied in the paper, can be applied both for plane systems and for space systems. In another future paper, also for this model, we will apply the method motions.

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